Mathematical and Computational Challenges in the Biological Sciences

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Abstract

The interface between mathematics, the computational sciences, and biology presents both opportunities and challenges. Opportunities have surfaced within the last three decades because of the enormous increase in the quantity and quality of biological data due to advances in technology and the availability of powerful computing power (hardware and software) that can potentially organize the plethora of biological data. However, further advances in this direction is necessary but may not be sufficient to further our understanding of complex biological systems. I will argue that there are two further requirements: (1) There is a need to integrate the information across different time and spatial scales, (2) There is a need for theoretical frameworks for approaching behaviour of spatially extended, hierarchical (multi-scale) systems.

Mathematical models do provide a theoretical framework but I will argue for new tools and conceptual developments to understand such complex dynamical systems. Part of the problem is the use of mathematical models to represent model structures and processes as different types of mathematical objects; for example, muscle fiber orientation is modelled by a tensor, while action potential in a cell can be modelled by solutions of differential equations. What is needed is a means of reconstructing the behaviour of a system from a detailed knowledge of its components and their interactions – given the baroque complexity of living systems any such reconstruction must be constructive and computational. For example, organismal biology deals with all aspects of the biology of individual plants and animals, including physiology, morphology, development, and behaviour. It interfaces cellular and molecular biology at one end, and ecology at the other. In cellular and molecular biology one attempts to develop integrative theories or organismal function and in ecology, one attempts to place individual behaviour and function within an environmental context.

There are several problems in understanding the behaviour of living systems even when a detailed and accurate description of its components is available: there is the sheer complexity of the system and the number of its components, the components operate over radically different time scales and spatial scales, the processes are occurring in a system that is spatially extended and organized within a structural and functional hierarchy, most of the functional interactions within biological systems are nonlinear, and small changes in the parameters of nonlinear systems can lead to large scale, qualitative changes in their behaviour. In this presentation I will highlight the developing synthesis of the nature of particular biological systems and the development of mathematical models and computational tools to understand and predict the behaviour of such systems. Towards this end, multi-scale modelling and computation, for example, is a rapidly evolving area of research that will have a fundamental impact on computational science and applied mathematics, and the understanding of biological systems.