Rubrique

OBSERVER DESIGN FOR A FISH POPULATION MODEL

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RÉSUMÉ. Le but de ce travail est d'appliquer des outils de contrôle aux systèmes de population de pêche. on construit un observateur pour un modèle continu structuré en age de population de pêche exploitée qui tient compte des pré-recrutés. Les variables du modèle, l'effort de pêche, les classes d'age et la capture sont considérés respectivement comme contrôleur, états du systèmes et sa sortie mesurée. Le changement de variables basé sur les dérivés de Lie nous a permis de mettre le système sous une forme canonique observable. La forme explicite de l'observateur est finalement donnée.

ABSTRACT. Our aim is to apply some tools of control to fishing population systems. In this paper one constructs a non linear observer for the continuous stage structured model of an exploited fish population, using the fishing effort as a control term, the age classes as a state and the quantity of caught fish as a measured output. Under some biological satisfied assumptions, we formulate the observer corresponding to this system and show its exponential convergence. With the Lie derivative transformation, one shows that the model can be transformed to a canonical observable form; then one gives the explicit gain of the estimation.

MOTS-CLÉS : Modèle Structural, Pêche, Observateur, Population Dynamique, Ecosystème.

KEYWORDS : Structural Model, Fish, Observer, Population Dynamics, Ecosystem.

1. INTRODUCTION

In fish population science, one evolves in a dubious world where the observation and the direct experimentation are practically impossible. The resources cannot be counted directly, except with acoustic method which is not generalized yet. It is thus necessary to estimate the stock abundance through available data, captured quantity and fishing effort. In literature the stock estimation state has received a less deal of attention, and some authors are interested in the observer synthesis for the fish population systems. Ouahbi and al [1] consider the discrete time model to develop a global observer which doesn't require any non linear transformation, and it doesn't depend on any expression of recruitment function. J.L Gouze et al [4] present a technic for the dynamic estimation of bounds and no-measurable variables of an uncertain dynamical systems. They show the applicability of these method only to the model of three stages. In this work we are interested in providing the estimation of the state for the model with n stages using the known input and the measured output, having recourse to some global results found out by Gauthier et al[6], Farza et al[8] .
The paper is organized as follows. We first consider the description of the continuous stage structured model, under some biological satisfied assumptions. Next we give a state transformation in order to make our system in a canonical observable form relying on the Lie derivative transformation. Then we investigate the technic for the estimation of the abundance in an invariant domain. In section 4, simulation results are shown for $n = 4$. Finally in section 5, a conclusion is given.

2. PROBLEM FORMULATION AND ASSUMPTIONS

One considers here the nonlinear model derived in [10] and which describes the fish population dynamics of abundance X_i and exploited by the fleet represented by the total catch Y and the fishing effort E.This model is described by the following state equation.

$$\left\{\begin{array}{lcl} \dot{X}_0 & = & -\alpha_0 X_0 + \sum_{i=1}^{n} f_i l_i X_i - \sum_{i=0}^{n} p_i X_i X_0 \\ \dot{X}_1 & = & \alpha X_0 - (\alpha_1 + q_1 E) X_1 \\ \vdots & & \vdots \\ \dot{X}_n & = & \alpha X_{n-1} - (\alpha_n + q_n E) X_n \\ Y & = & q_1 E X_1 + q_2 E X_2 + ... + q_n E X_n \end{array}\right. \qquad [1]$$

where p_0 and p_i represent respectively, the juvenile competition parameter and predation of class i on class 0.
f_i and l_i are, respectively, the fecundity rate, and reproduction efficiency of class i.
The natural mortality class rate i is M_i, and the relative catchability coefficient is q_i.
the linear aging coefficient α, is supposed to be constant. α_i is defined as $\alpha_i = \alpha + M_i$
.Suppose the system [1] satisfies assumptions as below :
Assumption 2.1 (one non linearity at least must be considered) $\sum_{i=0}^{n} p_i \neq 0$
Assumption 2.2 (the spawning coefficient must be big enough so as to avoid extinction) $\sum_{i=1}^{n} f_i l_i \pi_i > \alpha_0$ where $\pi_i = \frac{\alpha^i}{\prod_{j=1}^{i}(\alpha_j + q_j \bar{E})}$ and $\bar{E}$ is a constant fishing effort.
Assumption 2.3(all age classes are subject to catch and the oldest one yields eggs) for all $i = 1 \ldots n$ $q_i > 0$ and $f_n l_n \neq 0$
Assumption 2.4 (each predator lays more eggs than it consumes) $X_0 < \mu = \min_{i=1\ldots n}(\frac{f_i l_i}{p_i})$

for $f_i l_i p_i \neq 0$.
Assumption 2.5 The fishing effort is subject to the constraint : $0 < E_{min} \leq E \leq E_{max}$
The system [1] has two equilibrium points : the origin $X = 0$ corresponds to an extincted population and the nontrivial equilibrium X^*, w here $X_i^* = \pi_i X_0^*$ and $X_0^* = \frac{\sum_1^n f_i l_i \pi_i - \alpha_0}{p_0 + \sum_1^n p_i \pi_i}$
In [2] it was shown that the system [1] controlled by any positive constant feedback law $\overline{E}$ is asymptotically stable.
To facilitate the design of the observer the fishing effort is considered constant.

3. NON LINEAR OBSERVER DESIGN

3.1. State transformation

The system [1] can be rewritten with standard control notation :

$$\left\{ \begin{array}{lcl} \dot{X} & = & A1X + BXu + \zeta(X) \\ Y & = & C_1 X \end{array} \right. \quad [2]$$

$$where: \quad A1 = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ \alpha & 0 & 0 & \dots & 0 \\ 0 & \alpha & 0 & \dots & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \alpha & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & -q_1 & 0 & \dots & 0 \\ 0 & 0 & -q_2 & \dots & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & -q_n \end{bmatrix}$$

$$\zeta(X) = \begin{bmatrix} -\alpha_0 X_0 + \sum_{i=1}^n f_i l_i X_i - \sum_{i=0}^n p_i X_i X_0 \\ -\alpha_1 X_1 \\ \vdots \\ \alpha_n X_n \end{bmatrix} \qquad C_1 = [0, q_1 u, q_2 u, \dots, q_n u]$$

In order to get asymptotic results. We restrict Our Study to the Set D defined as follows $D = \Pi_1^n [a_i, b_i]$ where a_i can be chosen as small as one need and $b_i = (\mu + \upsilon_i)\pi_i$ with $\upsilon_0 = 0 < \upsilon_1 < ... < \upsilon_n < 1$. it is shown in [10] that a_i and b_i are bounded by some function of the parameter f_i,l_i and π_i and that D is an invariant domain by system [1]
Let us prove that ζ is liptshitz in D
By the mean value theorem there exist a point z on the line segment joining $X^1 \in D$ and $X^2 \in D$ such that :
$\zeta(X^1) - \zeta(X^2) = \frac{\partial \zeta}{\partial X}(Z)(X^1 - X^2)$
thus

$$\begin{array}{lcl} \|\zeta(X^1) - \zeta(X^2)\| & = & \|\frac{\partial \zeta}{\partial X}(Z)(X^1 - X^2)\| \\ & \leq & \|\frac{\partial \zeta}{\partial X}(Z)\| \|(X^1 - X^2)\| \\ & \leq & (2p_0\mu + \sum_1^n p_i b_i + \sum_1^n f_i l_i + (\alpha_0^2 + \alpha_1^2 \dots + \alpha_n^2)^{\frac{1}{2}}) \|X^1 - X^2\| \end{array}$$

So ζ is lipschitz in the invariant domain D with the liptschitz constant L=$2p_0\mu$+$\sum_1^n p_i b_i +$ $\sum_1^n f_i l_i + (\alpha_0^2 + \alpha_1^2 \dots + \alpha_n^2)^{\frac{1}{2}}$.
Let $f(X) = A_1 X$, and $g(X) = BX$
To facilitate the design of the nonlinear observer, perform a nonlinear state transformation : $\phi : X \longrightarrow Z = (h(X), L_f h(X), \dots, L_f^n h(X))$
L denotes the Lie derivative operator : $L_f h(X) = \frac{\partial h(X)}{\partial X} f(X)$ and $L_f^n h(X) = L_f L_f^{n-1} h(X)$

$Z = (Z_0, Z_1, ..., Z_n)$ can be expressed as : $Z = \phi(X) = MX$ where

$$M = \begin{bmatrix} 0 & q_1u & q_2u & \dots & q_nu \\ q_1u\alpha & q_2u\alpha & \dots & q_nu\alpha & 0 \\ q_2u\alpha^2 & \dots & q_nu\alpha^2 & 0 & 0 \\ \vdots & . & 0 & 0 & 0 \\ q_nu\alpha^n & 0 & 0 & 0 & 0 \end{bmatrix}$$

One shows easily that $\forall\ (u, q_n) \neq (0,0)$ $\det M = q_n^{n+1}u^{n+1}\alpha^{\frac{n(n+1)}{2}} \neq 0$
Thus ϕ is a Diffeomorphism in D
Having recourse to some global results found out by Gauthier et al[6] and Farza et al[8]
ϕ transform [2]to :

$$\begin{cases} \dot{Z} &= AZ + \psi(Z)u + \varphi(Z) + \omega(Z) \\ Y &= CZ \end{cases} \qquad [3]$$

Where

$$A = \begin{bmatrix} 0 & 1 & \dots & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = [1, 0, 0, \dots, 0] \qquad \varphi(Z) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ L_f^{n+1}h(\phi^{-1}(Z)) \end{bmatrix} = 0$$

$(L_f^{n+1}h(\phi^{-1}(Z)) = C_1A_1^{n+1}X = 0)$

$$\psi(Z) = \begin{bmatrix} L_gL_f^0h(\phi^{-1}(Z)) \\ L_gL_f^1h(\phi^{-1}(Z)) \\ \vdots \\ L_gL_f^nh(\phi^{-1}(Z)) \end{bmatrix} = BZ \qquad \begin{aligned} \omega(Z) &= \frac{\partial\phi}{\partial X}(\phi^{-1}(Z))\zeta(\phi^{-1}(Z)) \\ &= M\zeta(\phi^{-1}(Z)) \end{aligned}$$

ω is lipschitz in the invariant domain D with the constant L(ζ is liptschitz with the constant L)

3.2. Nonlinear Estimation Design

Our goal is to design an asymptotic state Observer $\widehat{X}$ with inputs $\overline{E}$ and Y its output, such that $\|\widehat{X} - X\|$ tends to zero as t goes to infinity.
Let S_θ the solution of the algebraic equation :$\theta S_\theta + A'S_\theta + S_\theta A - C'C = 0$
and d_θ be a definite diagonal matrix defined by : $d_\theta = diag(1, \frac{1}{\theta}, \dots, \frac{1}{\theta^n})$ where $\theta > 0$
it is proved that $S_\theta = \frac{1}{\theta}d_\theta S_1 d_\theta$[8]
where S_1 is the solution of the algebraic equation four $\theta = 1$ and $S_1(i,j) = (-1)^{i+j}C_{i+j-1}^{j-1}$
$S_1^{-1}C' = [C_{n+1}^1, C_{n+1}^n, \dots, C_{n+1}^{n+1}]'$

3.2.1. proposition

For θ large enough the dynamical system modeled by :

$$\dot{\widehat{X}} = f(\widehat{X}) + ug(\widehat{X}) + \zeta(\widehat{X}) - \theta M^{-1}d_\theta^{-1}S_1^{-1}C'(C_1\widehat{X} - Y) \qquad [4]$$

where $f(\widehat{X}) = A_1\widehat{X}$ and $g(\widehat{X}) = B\widehat{X}$ is an exponential observer for the system [2].

3.2.2. Lemma

For θ large enough the dynamical system modeled by : $\dot{\widehat{Z}} = A\widehat{Z} + \psi(\widehat{Z})u + \omega(\widehat{Z}) - \theta d_\theta^{-1} S_1^{-1} C'(C\widehat{Z} - Y)$ is an exponential observer for the system [3].

Proof of the Lemma

Let $e = \widehat{Z} - Z$
$\dot{e} = (A - \theta d_\theta^{-1} S_1^{-1} C'C)e + (\psi(\widehat{Z}) - \psi(Z))u + (\omega(\widehat{Z}) - \omega(Z))$
taking into account $\theta d_\theta^{-1} A d_\theta = A$ and $CC'd_\theta = C'C$ it follows
$\dot{e} = \theta d_\theta^{-1}(A - S_1^{-1}C'C)d_\theta e + (\psi(\widehat{Z}) - \psi(Z))u + (\omega(\widehat{Z}) - \omega(Z))$
Let $e_\theta = d_\theta e$
So $\dot{e_\theta} = \theta(A - S_1^{-1}C'C)e_\theta + d_\theta(\psi(\widehat{Z}) - \psi(Z))u + d_\theta(\omega(\widehat{Z}) - \omega(Z))$
Consider the lyapunov function defined as : $V(e_\theta) = e'_\theta S_1 e_\theta$
$\dot{V} = \theta(e'_\theta(S_1A + A'S_1)e_\theta - 2e'_\theta C'Ce_\theta) + 2e'_\theta S_1 d_\theta((\psi(\widehat{Z}) - \psi(Z))u + (\omega(\widehat{Z}) - \omega(Z)))$
$So\dot{V} = \theta e'_\theta(S_1A + A'S_1 - 2C'C)e_\theta + 2e'_\theta S_1 d_\theta((\psi(\widehat{Z}) - \psi(Z))u + (\omega(\widehat{Z}) - \omega(Z)))$
from the algebraic equation we obtain :
$\dot{V} = e'_\theta(-\theta S_1 - \theta C'C)e_\theta + 2e'_\theta S_1 d_\theta((\psi(\widehat{Z}) - \psi(Z))u + (\omega(\widehat{Z}) - \omega(Z)))$
Then $\dot{V} = -\theta V - \theta e'_\theta C'Ce_\theta + 2e'_\theta S_1 d_\theta((\psi(\widehat{Z}) - \psi(Z))u + (\omega(\widehat{Z}) - \omega(Z)))$
consequently $\dot{V} \leq -\theta V + 2\lambda_{max}(S_1)\sqrt{n+1}(L + L_m)\|e_\theta\|^2$ Where $L_m = \|B\|u_{max}$
thus $\dot{V} \leq (2\frac{\lambda_{max}(S_1)\sqrt{n+1}(L+L_m)}{\lambda_{min}(S_1)} - \theta)V$

By the Bellman-Gronwall lemma we deduce that :
$V(t) \leq V(0)\exp(-(\theta - 2\frac{\lambda_{max}(S_1)\sqrt{n+1}(L+L_m)}{\lambda_{min}(S_1)})t)$
So

$$\begin{aligned}
\|e_\theta(t)\| &\leq \sqrt{\frac{V(0)}{\lambda_{min}(S_1)}}\exp(-(\theta - 2\frac{\lambda_{max}(S_1)\sqrt{n+1}(L+L_m)}{\lambda_{min}(S_1)})\frac{t}{2}) \\
&\leq \sqrt{\frac{\lambda_{max}(S_1)}{\lambda_{min}(S_1)}}\|e_\theta(0)\|\exp(-(\theta - 2\frac{\lambda_{max}(S_1)\sqrt{n+1}(L+L_m)}{\lambda_{min}(S_1)})\frac{t}{2}) \\
&\leq \sigma(S_1)\|e_\theta(0)\|\exp(-(\theta - 2\frac{\lambda_{max}(S_1)\sqrt{n+1}(L+L_m)}{\lambda_{min}(S_1)})\frac{t}{2})
\end{aligned}$$

$(\sigma(S_1) = \sqrt{\frac{\lambda_{max}(S_1)}{\lambda_{min}(S_1)}})$
Using the following inequality :
$\frac{\|e(t)\|}{\theta^n} \leq \|e_\theta(t)\| \leq \|e(t)\|$
one deduces

$$\begin{aligned}
\|e(t)\| &\leq \theta^n\|e_\theta(t)\| \\
&\leq \theta^n\sigma(S_1)\exp(-(\theta - 2\frac{\lambda_{max}(S_1)\sqrt{n+1}(L+L_m)}{\lambda_{min}(S_1)})\frac{t}{2})\|e(0)\|
\end{aligned} \quad [5]$$

Thus
for all $\theta \geq 2\frac{\lambda_{max}(S_1)\sqrt{n+1}(L+L_m)}{\lambda_{min}(S_1)}$ $\|e(t)\|$ tends to zero
Which ends the proof of the lemma
Proof of the proposition
We have :
$\dot{\widehat{X}} = M^{-1}\dot{\widehat{Z}}$
Thus

$$\begin{aligned}
\dot{\widehat{X}} &= M^{-1}(A\widehat{Z} + \psi(\widehat{Z})u + \omega(\widehat{Z}) - \theta d_\theta S_1^{-1}C'(C\widehat{Z} - y)) \\
&= f(\widehat{X}) + g(\widehat{X})u + \zeta(\widehat{X}) - \theta M^{-1}d_\theta S_1^{-1}C'(C1\widehat{X} - y) \\
&= f(\widehat{X}) + g(\widehat{X})u + \zeta(\widehat{X}) - \theta M^{-1}d_\theta S_1^{-1}C'(C1\widehat{X} - Y)
\end{aligned}$$

We can prove that D is also invariant by the system [4]
However the gain $M^{-1}d_\theta^{-1}S_1^{-1}C'(C_1\widehat{X}-Y)$ of the observer [4] could be explicitly written as :

$$-\theta M^{-1}d_\theta^{-1}S_1^{-1}C'(C_1\widehat{X}-Y)=\begin{bmatrix} P_0(\theta)C_1(\widehat{X}-X) \\ P_1(\theta)C_1(\widehat{X}-X) \\ \vdots \\ P_n(\theta)C_1(\widehat{X}-X) \end{bmatrix}$$

$Where$P$_i(\theta)$ is a polynomial of degree n+1
From the inequality [5] we get $\lim_{\theta\longmapsto+\infty} P_i(\theta)C_1(\widehat{X}-X)=0$ So

$$\forall\epsilon_i>0\exists\theta_i>0\forall\theta>\theta_i\|P_i(\theta)C_1(\widehat{X}-X)\|<\epsilon_i \qquad [6]$$

By choosing appropriate ϵ_i we can find θ_i such that $\forall\theta>\theta_i$
$\dot{\widehat{X}}_i(a_0,a_1,\ldots,a_n)>0$ and $\dot{\widehat{X}}_i(b_0,b_1,\ldots,b_n)<0$
Then D is also invariant by the system (4)

3.3. observer for the model of three stages(n=2)

the equation of the model is expresed as :

$$\left\{\begin{array}{lcl} \dot{X}_0 &=& -\alpha_0X_0+\sum_{i=1}^2 f_il_iX_i-\sum_{i=0}^2 p_iX_iX_0 \\ \dot{X}_1 &=& \alpha X_0-(\alpha_1+q_1E)X_1 \\ \dot{X}_2 &=& \alpha X_1-(\alpha_2+q_nE)X_2 \\ Y &=& q_1EX_1+q_2EX_2 \end{array}\right. \qquad [7]$$

The observer is given as :

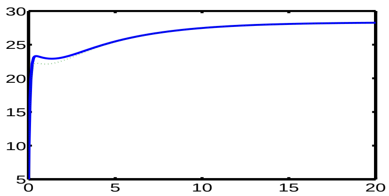
$$\left\{\begin{array}{lcl} \dot{\widehat{X_0}} &=& -\alpha_0\widehat{X_0}+\sum_{i=1}^2 f_il_i\widehat{X_i}-\sum_{i=0}^2 p_i\widehat{X_i}\widehat{X_0}+\frac{\theta^3}{\alpha^2}\frac{1}{q_2}(q_1\widehat{X_1}+q_2\widehat{X_2}-Y) \\ \dot{\widehat{X_1}} &=& \alpha\widehat{X_0}-(\alpha_1+q_1E)\widehat{X_1}+(3\frac{q_1\theta^2}{q_2\alpha}-3\frac{q_1^2\theta^3}{q_2^2\alpha^2})\frac{1}{q_2}(q_1\widehat{X_1}+q_2\widehat{X_2}-Y) \\ \dot{\widehat{X_2}} &=& \alpha\widehat{X_1}-(\alpha_2+q_2E)\widehat{X_2}+(3\theta-3\frac{q_1\theta^2}{q_2\alpha}+\frac{q_1^2\theta^3}{q_2^2\alpha^2})\frac{1}{q_2}(q_1\widehat{X_1}+q_2\widehat{X_2}-Y) \\ Y &=& q_1EX_1+q_2EX_2 \end{array}\right. \qquad [8]$$

4. SIMULATION RESULTS AND DISCUSSION

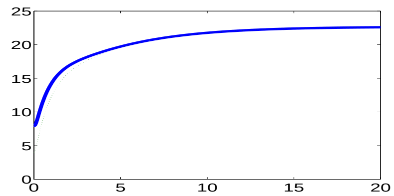
One considers here a population with five stages age (n=4) :
Stage 0 represents the abundance of juvenile ; stage 1 represents the young adults abundances without reproduction and cannibalism ; the stages 2,3 and 4 are adults abundances with the same term of predation and the same proportion on the female mature but have different reproduction rate ($l_2\leq l_3\leq l_4$).).
The results obtained from the observer are illustrated by the example characterized by the parameter value inspired from literature data [10]given in table1. one simulates two cases with the input $E(t)=\overline{E}$. In order to show the effect of high gain, we first simulate the proposed system with the high gain $\theta=5$ and the results are presented in figures 1 which give time evolution of the stage age X_i and theirs estimates $\widehat{X_i}$ respectively for $i=0$ to 2 . Then in Figures2 we give the simulation results with the high gain $\theta=15$. Both the two values of θ guarantees asymptotic convergence, and the second one shows good tracking performances than the first.

stage i	0	1	2	3	4	stage i	0	1	2	3	4
p_i	0.2	0	0.1	0.1	0.1	M_i	0.5	0.2	0.2	0.1	0.05
f_i			0.5	0.5	0.5	α			0.8		
l_i		0	10	20	15	α_i	1.3	1	1	0.9	0.85
m_i	0.5	0.2	0.2	0.2	0.2	$\bar{E}$			1		
q_i	0	0	0	0.1	0.15	X_{ini}	5	8	10	10	8
$\widehat{X}_{ini}$	6	4	5	10	8						

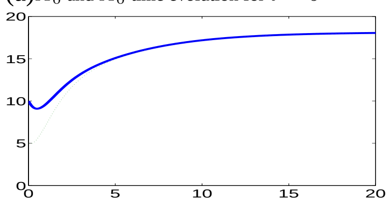
Tableau 1. *simulation data*



(a)X_0 and $\widehat{X_0}$ time evolution for $\theta = 5$

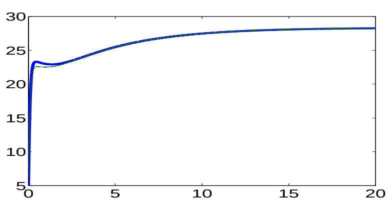


(b)X_1 and $\widehat{X_1}$ time evolution for $\theta = 5$

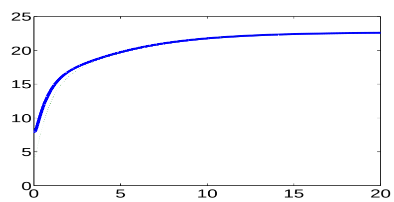


(c)X_2 and $\widehat{X_2}$ time evolution for $\theta = 5$

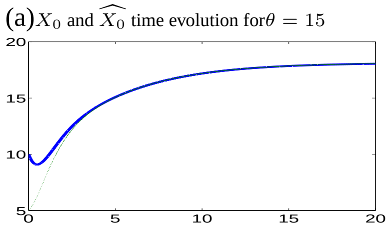
Figure 1. *Convergence asymptotic of the observer with the high gain $\theta = 5$ ("_ ") corresponds X_i and ("...") corresponds to $\widehat{X}_i$*



(a)X_0 and $\widehat{X_0}$ time evolution for$\theta = 15$



(a)X_1 and $\widehat{X_1}$ time evolution for $\theta = 15$



(c)X_2 and $\widehat{X_2}$ time evolution for $\theta = 15$

Figure 2. *Convergence asymptotic of the observer with the high gain $\theta = 15$ ("_ ") corresponds X_i and ("...") corresponds to $\widehat{X}_i$*

5. CONCLUSION

We are interested in constructing a simple observer for the harvested fish population model structured in n ages classes, in an invariant domain using the Lie Derivative transformation. The asymptotic state observer is explicitly formulated.

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