Rubrique

ON THE STABILIZATION OF A SYSTEM DESCRIBING THE DYNAMICS OF A FISHERY

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RÉSUMÉ. Le but de ce travail est d'appliquer des outils de contrôle au Modèle continu structuré en age de population de pêche exploitée, qui tient compte des pré-recrutés, on construit une commande linéaire par retour d'état qui permet de stabiliser le système autour d'un point de fonctionnement. L'effort de pêche, les classes d'age et la capture sont considérés respectivement comme contrôleur, états du système et sa sortie mesurée. L'èquation de lyaponov et celle de Riccati sont utulisèes et permettent de dèterminer le domaine de stabilité.

ABSTRACT. In this paper one uses some tools of automatic control to stabilize the continuous age-structured fishery model. More precisely one takes the fishing effort as a control term the age classes as a states and the total caught as a measured output. One interests in the design of a fishing strategy by constructing a linear feedback control low that permits to stabilize the studied system around a nontrivial steady state. The Lyapunov equation and the Riccati equation are used to construct the control low and permit to indicate the domain of stability.

MOTS-CLÉS : Modèle Structural Continu, Pêche, Stabilisation, Retour d'état, Population Dynamique, Ecosystème.

KEYWORDS : Continuous Structured Model, Fish, Linear Feedback Control, Population Dynamics, Ecosystem

1. INTRODUCTION

More than two thirds of the surface of the earth are covered by oceans or seas. As results fish and other marine products form an important source of food. Fishing technology has developed for small boats to swimming fishing factories with sophisticated equipment for detecting, catching and processing fish. The high demand for food and in particular protein food causes the increase of the fishing effort, and the fish stocks is over-exploited, resulting in large scale of fisheries closure. So in this age of environmental crises one needs to meet the challenge of managing resources fish stocks.
Several researches are realized in fish population system modeling in order to describe qualitatively and quantitatively various fisheries events, to predict future evolution of a given population, to explore a fishing strategies and to maintain fish stocks at levels sufficient to produce maximum sustainable yields(MSY). Then many dynamic models were made to represent the stock : Global models[3,4],where the individuals are aggregated in an unique variable, and Structured models[1,2,5,7,8] which distinguishes between several stages(age class, stages...) of the stock. The structured models are the most representative of the harvested stocks, it takes into account several parameters. The control of this fish population models has received a less deal of attention. In the literature we find the work of ([3],[7], [2],[9],[6]). In [2] the authors design a fishing strategy in order to regulate the exploited fish population, they used a stage-structured model, and give a formula for the fishing effort as a nonlinear feedback control that allows to stabilize the system. They also prove that a constant fishing effort stabilize the studied system. A constant control has not the possibility of fighting the effect of the perturbations except if those remain constant. For that and in order to automate the continuous fishery process a linear state feedback control is the most appropriate because it's easy to realize in practice with the PID controller. The interest of Our work is to construct a linear feedback control to stabilize the harvested continuous stage structured fishery system, having recourse to some global results found out by zak [10]. one computes the controller in such a way that the closed loop system with this feedback has a nontrivial equilibrium state which is locally asymptotically stable. The Lyapunov equation and the Riccati equation are used to construct the control low and permit to indicate the domain of stability.

The paper is organized as follows, one first presents the fishery model and its properties. Then one concentrates on the regulation process. Next a numerical example is chosen to illustrates the methods and the simulation results are shown. Finally a conclusion and perspective work are exposed.

2. PRESENTATION OF THE FISHING MODEL

One considers here the nonlinear model derived in [7] and which describes the fish population dynamics of abundance X_i and exploited by the fleet represented by the total catch Y and the fishing effort E. This model is described by the following state equation.

$$\left\{ \begin{array}{lcl} \dot{X}_0 & = & -\alpha_0 X_0 + \sum_{i=1}^{n} f_i l_i X_i - \sum_{i=0}^{n} p_i X_i X_0 \\ \dot{X}_1 & = & \alpha X_0 - (\alpha_1 + q_1 E) X_1 \\ \vdots & & \vdots \\ \dot{X}_n & = & \alpha X_{n-1} - (\alpha_n + q_n E) X_n \\ Y & = & q_1 E X_1 + q_2 E X_2 + ... + q_n E X_n \end{array} \right. \qquad [1]$$

where

p_0 and p_i represent respectively, the juvenile competition parameter and predation of class i on class 0.
f_i and l_i are respectively the fecundity rate and reproduction efficiency of class i.
M_i is the natural mortality class rate i , and q_i the relative catchability coefficient.
the linear aging coefficient α is supposed to be constant and defined as : $\alpha_i = \alpha + M_i$

All the parameters of the model are positive. The recruitment from one class to another can be represented by a strictly positive coefficient of passage. The passage rate α from the juvenile class to the adult stages is supposed to be constant with respect to time and stages. This means that the time of residence is equal to $\frac{1}{\alpha}$. The laying eggs is considered continuous with respect to time. The total number of eggs introduced in the juvenile stage is given by $\sum_1^i f_i l_i X_i$. The cannibalism term $\sum_1^i p_i X_0 X_i$ is based on the Lotka-Volterra predating term between class i and class 0. The intra-stage competition for food and space is expressed as $p_0 X_0^2$. The mortality of each stage i is caused by the fishing and natural mortality which is supposed linear.

One supposes that the system (1) satisfies the following assumptions :

Assumption 2.1 (one non linearity at least must be considered) $\sum_{i=0}^n p_i \neq 0$

Assumption 2.2 (the spawning coefficient must be big enough so as to avoid extinction) $\sum_{i=1}^n f_i l_i \pi_i > \alpha_0$ where $\pi_i = \frac{\alpha^i}{\prod_{j=1}^i (\alpha_j + q_j \bar{E})}$ and $\bar{E}$ is a constant fishing effort.

Assumption 2.3(all age classes are subject to catch and the oldest one yields eggs)
for all $i = 1 \ldots n$ $q_i > 0$ and $f_n l_n \neq 0$

Assumption 2.4 (each predator lays more eggs than it consumes) $X_0^* < \mu = \min_{i=1\ldots n}(\frac{f_i l_i}{p_i})$ for $f_i l_i p_i \neq 0$.

The system (1) has two equilibrium points : the first one is the origin $X = 0$ which corresponds to an extinct population and is therefore not very interesting. the second one is the nontrivial equilibrium X^* defined as :
$X_0^* = \frac{\sum_1^n f_i l_i \pi_i - \alpha_0}{p_0 + \sum_1^n p_i \pi_i}$
$X_i^* = \pi_i X_0^*$

3. LINEAR STATE FEEDBACK CONTROL

3.1. State Transformation

Using the change of coordinate $x_i = X_i - X_i^*$ and $u = E - \overline{E}$ The System (1) can be transformed into :

$$\dot{x} = Ax + Bu + \xi(x, u) \ . \quad [2]$$

where

$$A = \begin{bmatrix} k_0 & k_1 & k_2 & \dots & k_n \\ \alpha & -(\alpha_1 + q_1\overline{E}) & 0 & 0 & 0 \\ 0 & \alpha & -(\alpha_2 + q_2\overline{E}) & 0 & 0 \\ \vdots & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \alpha & -(\alpha_n + q_n\overline{E}) \end{bmatrix}$$

$k_0 = -(\alpha_0 + 2p_0X_0^* + \sum_{i=0}^{n} p_iX_i^*)$ and $k_i = l_if_i - p_iX_0^*$

$$B = \begin{bmatrix} 0 \\ -q_1X_1^* \\ -q_2X_2^* \\ -q_3X_3^* \\ \vdots \\ -q_nX_n^* \end{bmatrix} \quad \xi(x,u) = Cxu + g(x,u)$$

C=$diag(0, -q_1, -q_2, \cdots, -q_n)$

and $g(x,u) = \begin{bmatrix} -\sum_0^n p_ix_0x_i, & 0, & 0, & \dots, & 0 \end{bmatrix}'$

3.2. Stabilization Via Lyapunov Function

Let $M_n = -A$ and Δ_n=detM_n, we can easily prove that all pricipal minors of M_n are strictly positive.

By developing according to the first line Δ_n can written as ;

$\Delta_n = -\prod_{j=1}^{n} \alpha_j \Sigma_{i=0}^{n} k_i\pi_i$

Taking into account $\Sigma_{i=1}^{n} k_i\pi_i = \alpha_0 - \Sigma_{i=1}^{n} f_il_i\pi_i$ we get

$\Delta_n = -\prod_{j=1}^{n} \alpha_j(\alpha_0 - \Sigma_{i=1}^{n} f_il_i\pi_i)$

The assumption 2.2 implies that $\Delta_n > 0$

So the principal minors of M_n are expressed as :

$\Delta_q = -\prod_{j=1}^{q} \alpha_j \Sigma_{i=0}^{q} k_i\pi_i > 0$ where $q \in [1, n]$

Then M_n is a M-matrix and all it's eigenvalues are with real part strictly positive.
Consequently all eigenvalues of A with real part strictly negative
Finally A is asymptotically stabe

Then the solution P of the lyapunov matrix equation $A'P + PA = -2Q$ exists for a real symmetric positive definite Q.

3.2.1. Proposition

For any constant positive γ the system (2) controlled by the following controller $u = -\gamma B'Px$ is asymptotically stable in the Domain D defined as : D={x $\in R^{n+1}/ \|x\| \leq \frac{\lambda_{min}(Q)+\gamma\lambda_{min}(PBB'P)}{\|P\|(\gamma\|PB'\|\|q\|+\|p\|)}$}.

Proof

Assume we applied a stabilizing feedback $u = -\gamma B'Px$ to the system (2) :

Let V the following candidate lyapunov function :

$$V(x) = x^\top Px \qquad [3]$$

the time derivative of (3)is :

$$\begin{array}{rcl} \frac{dV(x)}{dt} & = & x'(A'P+PA)x+2x'PBu+2x'P\xi(x) \\ & = & -2x'Qx-2\gamma x'PBB'Px+2x'P\xi(x) \end{array} . \quad [4]$$

So
$\frac{dV(x)}{dt} \leq -2(\lambda_{min}(Q)+\gamma\lambda_{min}(PBB'P))\|x\|^2+2\|x\|\|P\|\|\xi(x)\|$
Let $q=(0,q_1,q_3,\ldots,q_n)$ and $p=(p_0,p_1,p_2,\ldots,p_n)$ ($\|C\|=\|q\|$)
Taking into account

$$\|\xi(x)\| \quad \leq \quad (\gamma\,\|PB'\|\|q\|+\|p\|)\|x\|^2 \quad [5]$$

It follows
$\frac{dV(x)}{dt} \leq -2(\lambda_{min}(Q)+\gamma\lambda_{min}(PBB'P)-\|P\|\|x\|(\gamma\,\|PB'\|\|q\|+\|p\|))\|x\|^2$
Finally $\dot{V} \leq 0$ if $\|x\| \leq \frac{\lambda_{min}(Q)+\gamma\lambda_{min}(PBB'P)}{\|P\|(\gamma\|PB'\|\|q\|+\|p\|)}$

3.3. Stabilization Via Riccati Equation

Let P_1 the solution of the the Riccati equation
$A'P_1+P_1A-P_1BR^{-1}B'P_1+Q_1=0$ where Q_1 is a real symetric positive definite matrix. R is a positive scalair. Assume now that the controller $u=-\gamma B'P_1x$ where $\gamma=R^{-1}$ sabilize the system (2) :

3.3.1. Proposition

For any constant positive γ the system (2) controlled by the following controller $u=-\gamma B'P_1x$ is asymptotically stable in the Domain D_1 defined as : $\mathrm{D}_1=\{x\in R^{n+1}/\|x\| \leq \frac{\lambda_{min}(Q_1)+\gamma\lambda_{min}(P_1BB'P_1)}{2\|P_1\|(\gamma\|P_1B'\|\|q\|+\|p\|)}\}$

Proof
Let V_1 the following candidate lyapunov function :

$$V_1(x) \quad = \quad x^\top P_1 x \quad [6]$$

the time derivative of (6)is :

$$\begin{array}{rcl} \frac{dV_1(x)}{dt} & = & x'(A'P_1+P_1A)x+2x'P_1Bu+2x'P_1\xi(x) \\ & = & x'(A'P_1+P_1A)x-2\gamma x'P_1BB'P_1x+2x'P_1\xi(x) \\ & = & x'(A'P_1+P_1A-\gamma P_1BB'P_1)x-\gamma x'P_1BB'P_1x+2x'P_1\xi(x) \\ & = & -x'Q_1x-\gamma x'P_1BB'P_1x+2x'P_1\xi(x) \end{array} . \quad [7]$$

Then
$\frac{dV_1(x)}{dt} \leq -(\lambda_{min}(Q_1)+\gamma\lambda_{min}(P_1BB'P_1))\|x\|^2+2\|x\|\|P_1\|\|\xi(x)\|$
Taking into account (5) it follows
$\frac{dV_1(x)}{dt} \leq -(\lambda_{min}(Q_1)+\gamma\lambda_{min}(P_1BB'P_1)-2\|P_1\|\|x\|(\gamma\,\|P_1B'\|\|q\|+\|p\|))\|x\|^2$
Finally $\dot{V}_1 \leq 0$ if $\|x\| \leq \frac{\lambda_{min}(Q_1)+\gamma\lambda_{min}(P_1BB'P_1)}{2\|P_1\|(\gamma\|P_1B'\|\|q\|+\|p\|)}$

4. SIMULATION RESULTS AND DISCUSSION

One considers here a population with five stages age (n=4) :
Stage 0 represents the abundance of juvenile ; stage 1 represents the young adults abundances without reproduction and cannibalism ; the stages 2,3 and 4 are adults abundances

with the same term of predation and the same proportion on the female mature but have different reproduction rate ($l_2 \leq l_3 \leq l_4$).
The table1 presents the parameter values of the studied system, these parameters are inspired from literature data [7].

stage i	0	1	2	3	4	stage i	0	1	2	3	4
p_i	0.2	0	0.1	0.1	0.1	M_i	0.5	0.2	0.2	0.1	0.05
f_i			0.5	0.5	0.5	α			0.8		
l_i		0	10	20	15	α_i	1.3	1	1	0.9	0.85
m_i	0.5	0.2	0.2	0.2	0.2	$\bar{E}$			10		
q_i	0	0	0	0.1	0.15	x_{ini}	0	0.5	-1	1	-2
						X^*	5.71	4.57	3.66	1.54	0.52

Tableau 1. *table1*

The parameters of the controller are selected as :

$$P = \begin{bmatrix} 10.7888 & 6.3311 & 4.3539 & 2.9791 & 1.5735 \\ 6.3311 & 6.0649 & 4.1675 & 1.9715 & 0.8466 \\ 4.3539 & 4.1675 & 4.3340 & 1.7395 & 0.6176 \\ 2.9791 & 1.9715 & 1.7395 & 1.2587 & 0.3532 \\ 1.5735 & 0.8466 & 0.6176 & 0.3532 & 0.5458 \end{bmatrix} \quad Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 0.4476 & 0.4019 & 0.2309 & 1.2472 & 0.7016 \\ 0.4019 & 1.8360 & 1.1324 & 1.9415 & 0.9514 \\ 0.2309 & 1.1324 & 2.0924 & 2.0454 & 0.8626 \\ 1.2472 & 1.9415 & 2.0454 & 7.7197 & 3.7671 \\ 0.7016 & 0.9514 & 0.8626 & 3.7671 & 2.3687 \end{bmatrix} \quad Q_1 = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

γ	$\lambda_{min}(Q)$	$\lambda_{min}(Q_1)$	$\lambda_{min}(PBB'P)$	$\lambda_{min}(P_1BB'P_1)$	$\|PB\|$	$\|P_1B\|$	$\|P\|$	$\|P_1\|$
1	1	2	0.63	3.145	0.79	1.77	18.92	11.1

Tableau 2. *table2*

It is clear that the parameters satisfy assumptions (2.1), (2.2)and (2.3).
The obtained results are shown in figures (1) and (2) ; the first one presents the states time evolution with the lyapunov equation and the second one shows the states compentes convergence with ricccati equation.
The origin is locally stable by the too low controllers $u = -\gamma B'Px$ and $u_1 = -\gamma B'P_1x$ and the second one permets to widen the domain of stability. These domains are respectivelly determined as :
D={x$\in R^5/\|x\| \leq 0.152$} and D$_1 = \{x \in R^5/\|x\| \leq 0.31\}$
Figure1(d) and Figure2(d) gives the asymptotic convergence of u andu_1 to zero what does mean that the fishing effort E converge to constant fishing effort $\overline{E}$. Results in figure1(a,b,c)and figure2(a,b,c)shows the asymptotic convergence of the states to zero. This results means that X converge asymptotically to $X^* = (5.71, 4.57, 3.66, 1.54, 0.52)$.

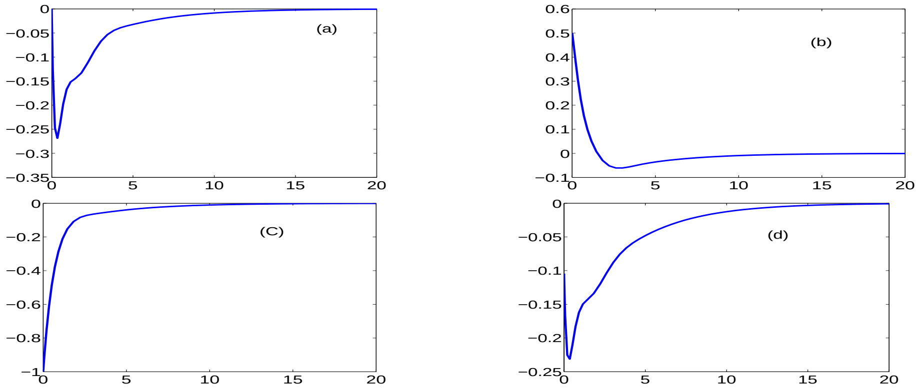

Figure 1. *The time evolution of the states x and the control low u construct by lyapunov equation (a),(b)and(c) represent respectivelly the states x_0,x_1 and x_2 ; (d)represents the control.*

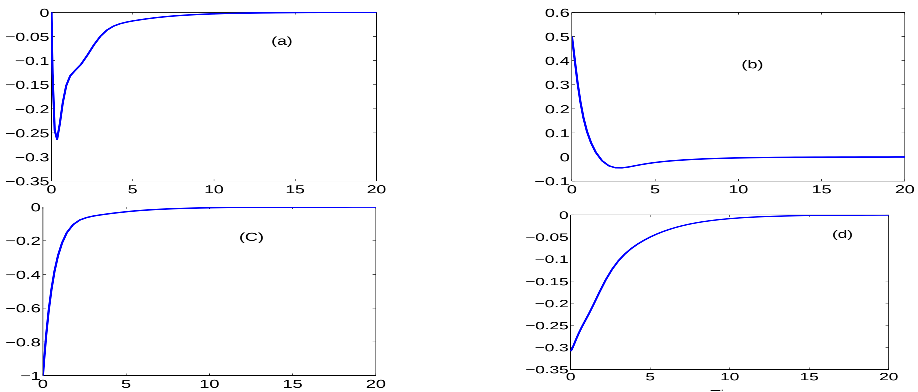

Figure 2. *The time evolution of the states x and the control low u construct by Riccati equation (a),(b)and(c) represent respectivelly the states x_0,x_1 and x_2 ; (d)represents the control.*

5. CONCLUSION

It is shown that one can regulates the fish stock in order to ensure the continuity of the population. A linear feedback control is designed to stabilize the harvested fish population system. One proves that the nontrivial equilibrium state is asymptotically stable. The lyapunov equation and the riccati equation are used to construct the control low and permit to indicate a domain of stability. one considers as example a simple case, but this regulator can be used with all species if the data are available. This work doesn't claim to solve the problem of fish stock stability but it is a first step towards the determination of a complete solution. The ongoing research work focuses on the stability and the estimation of the states when stock is subjected to environmental fluctuations and disturbances.

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