Timed Model-Checking Game

Case of Event-Recording Logic

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ABSTRACT. We consider the model-checking problem for a weak real-time logic called event-recording logic. We propose a two players parity game which decides if a real-time system modelled with an event-recording automaton is a model of a given formula φ. Region abstraction is done on the positions of the game in order to compute a winning strategy in that game. We show that a real-time system is a model of a formula if there is a winning strategy for one player in the game.

RÉSUMÉ. Nous traitons le problème de model-checking pour une logique temps réelle faible appelée event-recording logic. Les systèmes temps réels considérés sont modélisés à l'aide des systèmes de transition temporisés sans mémoires qui dérivent des event-recording automata. Nous proposons un jeu de parité à deux joueurs et nous montrons qu'un système est un modèle de notre formule s'il existe une stratégie gagnante pour un joueur déterminé dans le jeu. Le calcul d'une stratégie gagnante est rendu possible par l'abstraction de l'espace des positions du jeu à l'aide des régions.

KEYWORDS : model-checking, event-recording logic, parity game, event-recording automata

MOTS-CLÉS : verification, event-recording logic, jeux de parité, event-recording automata

1. Introduction

Sorea proposed [1] a decidable logic called Event-recording logic (ERL). ERL is timed extension of the mu-calculus [9]. In this paper, model-checking problem [8] of this logic is solved. Event-recording timed transition systems (ERTTS) are used to model real-time systems. ERTTS definition derives from the one of event-recording automata [5]. ERTTS and ERL are interesting for the proposed game in the way that we do not care about the clocks to be reseted. The local time-context model-checking problem solved in this paper is the following: Given an ERTTS $\mathcal{P}$ and an ERL formula φ, does the initial state p^0 of $\mathcal{P}$ satisfy φ in an initial time-context r^0. Game approach is used here to solve that problem.

We propose a two players parity game $\mathcal{G}(\mathcal{P}, \varphi, r^0)$. To check if p^0 satisfy φ in an initial time-context r^0 is reduced to checking the existence of a winning strategy for the first player in that game. Since real-time systems may have a infinite state space, an abstraction which uses regions is done on the arena the game. The regions feature ensures a finite number of positions in the game. This permits to obtain a finite parity game [10, 2].

Our method is similar to the one in [6, 4] which also inspired some of our definitions and results. The second part of the next section recalls some definitions. This includes definitions for ERTTS and ERL formulas; and the relation between a formula and an ERTTS. We define and we solve the model-checking game in section three. The last section concludes and discusses applicability of our results.

2. Definitions

2.1. Clocks, guards, valuations, regions

Let $\Sigma = \{a, a_0, a_1, \ldots, a_n\}$ be a set of events, $h(a_i)$ the clock associated to a_i, $\mathcal{H} = \{h(a_i) \mid a_i \in \Sigma\}$. A valuation $v : \mathcal{H} \mapsto \mathbb{R}_{\geq 0}$ assigns a positive real-value to each clock; $v(h)$ is the value the clock h. For $t \in \mathbb{R}_{\geq 0}$, the time's elapse's operation on v denoted $v + t$ is defined by $(v + t)(h) = v(h) + t$; $\mathtt{reset}(v, \{h\})$ is the valuation defined by $\mathtt{reset}(v, \{h\})(h(a)) = 0$ if $h(a) = h$ else, it is equal to $v(h(a))$.

Let $c \in \mathbb{Q}$ and $\sim \in \{<, >, \leq, \geq, =\}$, a time-guard g is generated by the grammar :
$g \ ::= tt \,|\, \mathit{ff} \,|\, h \sim c \,|\, h_1 - h_2 \sim c | g_1 \wedge g_2$.

The v-semantics of a time-guard g denoted $\|g\|$ is the set of valuations which satisfy g. The satisfaction relation $\models$ between a valuation v and a time-guard g is defined as follows : $v \models h \sim c$ iff $v(h) \sim c$, $v \models h_1 - h_2 \sim c$ iff $v(h_1) - v(h_2) \sim c$, $v \models g_1 \wedge g_2$ iff $v \models g_1$ and $v \models g_2$. Formally, $\|g\| = \{v \mid v \models g\}$.

The granularity of a set of time-guards $G = \{g_1, g_2, \ldots, g_n\}$ the tuple $gr = \langle H, m, \max\rangle$ where, H is a finite set of clocks, $m \in \mathbb{N}$ and $\max : H \mapsto \mathbb{Q}_{\geq 0}$. G is said gr-granular and, all the time-guards are gr-granular; that is, each time-guard $g \in G$ uses the clocks in H, each rational constant appearing in g is an integral multiple of m, and each clock $h \in \mathcal{H}$ is never compared to a constant larger than $\max(h)$.

A region r is an equivalence class of the valuations. Let $\mathtt{ent}(v_1(h))$ and $\mathtt{fract}(v(h))$ be the two functions which return respectively the integer and the fractional par of $v(h)$. The equivalence relation $v_1 \simeq v_2$ is defined by:

– For all $h \in \mathcal{H}$, either $\mathtt{ent}(v_1(h)) = \mathtt{ent}(v_2(h))$, or both $\mathtt{ent}(v_1(h))$ and $\mathtt{ent}(v_2(h))$ are greater than $\max(h)$.

– For all h_1,h_2 $\in \mathcal{H}$ with $v_1(h_1) \leq \max(h_1)$ and $v_1(h_2) \leq \max(h_2)$, $\mathtt{fract}(v_1(h_1)) \leq \mathtt{fract}(v_1(h_2))$ iff $\mathtt{fract}(v_2(h_1)) \leq \mathtt{fract}(v_2(h_2))$.

– For all $h \in \mathcal{H}$ with $v_1(h) \leq \max(h)$, $\mathtt{fract}(v_1(h)) = 0$ iff $\mathtt{fract}(v_2(h)) = 0$

We use $[v]$ to represent the equivalence class of v. $\mathcal{R}eg$ represents the set of all the regions.

Lemma 2.1 *[5] There are at most* $|\mathcal{H}|! \times 2^{|\mathcal{H}|} \times \Pi_{h\in\mathcal{H}}(2 \times \max(h) + 2)$ *regions.*

The semantics of a time-guard can be extended to regions. We say that $r \models g$ if there is at least one valuation in r which satisfies g. Because a region can be represented by using the time-guards, it is also convenient to say that $r \models g$ if $r \wedge g \neq$ *ff*. The time's elapse's operation on a region r denoted $r + t$ is defined by $(r + t) = \{[v + t] \mid v \in r\}$. The reset operation is defined by $\mathtt{reset}(r, \{h(a)\}) = \{[\mathtt{reset}(v, \{h(a)\})] \mid v \in r\}$. We remark that, when time elapses in a region, we may get a finite number of regions.

2.2. Event-recording time transition system

An event-recording time transition system (**ERTTS**) on Σ and $\mathcal{H}$ is a tuple $\mathcal{P} = \langle P, p^0, \rightarrow_{\mathcal{P}}\rangle$ where P is the set of control states of $\mathcal{P}$, p^0 is the initial state, and $\rightarrow_{\mathcal{P}} \subseteq P \times \Phi(\mathcal{H}) \times \Sigma \times P$ is the transition relation of $\mathcal{P}$. (p, g, a, p') represents a transition from the control state p to the the control state p'. We additionally require that $(p, g_1, a, p_1) \in \rightarrow_{\mathcal{P}}$ and $(p, g_2, a, p_2) \in \rightarrow_{\mathcal{P}}$ implies $g_1 \wedge g_2 =$ *ff*. Shortly (p, g, a, p') is called a (g, a)-transition from p to p'.

An ERTTS is gr-granular if the set of the guards of its transitions are gr-granular. In what follows, the considered ERTTS are gr-granular for some granularity gr.

The semantics of $\mathcal{P}$ is the finite transition system $\mathcal{S} = \langle S, s^0, \rightarrow_S\rangle$ where $S \subseteq P \times \mathcal{R}eg$ is the set of states also called configurations, $s^0 = (p^0, r^0)$ is the initial configuration, r^0 the initial region and $\rightarrow_S \subseteq S \times \Sigma \times S$ is the transition relation. There is a real-time transition $((p, r), a, (p', r')) \in\rightarrow_S$ if there is $(p, g, a, p') \in \rightarrow_{\mathcal{P}}$, $t \in \mathbb{R}_{>0}$ such that $r + t \models g$ and $r' \in \mathtt{reset}((r + t) \wedge g, \{h(a)\})$.

2.3. Event-recording logic

Let g represent a time-guard, a an event, $Var = \{X, Y, \ldots\}$ a set of variables. A formula φ of ERL [1] is generated by the following grammar:

$$\varphi ::= tt \,|\, \mathit{ff} \,|\, X \,| \varphi_1 \wedge \varphi_2 \,|\, \varphi_1 \vee \varphi_2 \,|\, [g, a]\varphi \,|\, \langle g, a\rangle\varphi \,|\, \mu X.\varphi \,|\, \nu X.\varphi$$

A variable X is bounded in φ if there is a sub-formula $\sigma X.\psi(X)$ of φ with $\sigma \in \{\nu, \mu\}$. $\sigma X.\psi(X)$ is the binding formula of X. An ERL formula φ is well named if each variable has a unique binding formula and, for each binding formula $\sigma X.\psi(X)$ in φ, the variable always X occurs in $\psi(X)$ in the scope of a modality ([] or $\langle\rangle$). If $\varphi(X)$ is a formula, $\varphi(Y/X)$ is obtained from $\varphi(X)$ by replacing X by Y. From now only well-named formulas are considered.

Given a formula φ, $Bd_\varphi(X)$ represents the binding formula of X in φ. X is a μ-variable if $Bd_\varphi(X) = \mu X.\psi(X)$ and X is a ν-variable if $Bd_\varphi(X) = \nu X.\psi(X)$.

New constants called binding constants and ranged over $U, V, \ldots$ are introduced and associated to each variable. The binding constant U associated to the variable X is defined by $U = Bd_\varphi(X)$. U is a μ-constant if it represents a μ-fix-point formula. U is a ν-constant if it represents a ν-fix-point formula. `Cons` represents the set of constants.

The definition list $\mathcal{D}(\varphi) = (U_1 = \sigma X_1.\varphi_1(X_1), U_2 = \sigma X_2.\varphi_1(X_2), \ldots, U_n = \sigma X_n.\varphi_1(X_n))$ of φ is a finite sequence of tuples constructed by means of $DL(\varphi)$ as follows:

– $DL(tt) = DL(\mathit{ff}) = DL(X) = DL(U) = \emptyset$

– $DL(\varphi_1 \vee \varphi_2) = DL(\varphi_1 \wedge \varphi 2) = DL(\varphi_1) \circ DL(\varphi_2)$

– $DL(\langle g, a\rangle\psi) = DL([g, a]\psi) = DL(\psi)$

– $DL(\mu X.\varphi(X)) = (U = \mu X.\varphi(X), DL(\varphi(U/X)))$ and U is a new constant

– $DL(\nu X.\varphi(X)) = (U = \nu X.\varphi(X), DL(\varphi(U/X)))$ and U is a new constant.

The operator $\circ$ concatenates two definition lists is such a way that no constant is defined twice.

For a formula φ and a definition list $\mathcal{D}$, the expansion operation $\mathtt{exp}_{\mathcal{D}}(\varphi)$, which subsequently replaces definition constants appearing in the formula by the right hand-sides of the defining equations, is defined as follows:

$$\mathtt{exp}_{\mathcal{D}}(\varphi) = \varphi(U_n/\psi_n)\ldots(U_1/\psi_1)$$

where $\mathcal{D} = ((U_1 = \psi_1), \ldots, (U_n = \psi_n))$ and $\psi_i = \sigma_i X_i.\varphi_i(X_i)$ and $\sigma_i \in \{\mu, \nu\}$.

Consider a formula $\sigma_i X.\varphi(X)$ and its sub-formula $\sigma_j Y.\varphi(Y)$. The variable Y depends on the variable X (denoted $X < Y$), if X occurs free in $\sigma_j Y.\varphi(Y)$.Given φ and $DL(\varphi)$, we say that U is older than V (denoted by $U \preceq V$) if U appears before V in $DL(\varphi)$. $\preceq$ is a total order on the binding constants of φ.

2.4. Relation between ERL and ERTTS

The semantics $\|\varphi\|_{\mathcal{P}}^{Val}$ of a formula φ with respect to an ERTTS $\mathcal{P}$ and an assignment $Val : \mathtt{Var} \mapsto 2^{P\times\mathcal{R}eg}$ is the set of configurations which satisfy φ. $Val[X/T]$ is an assignment defined such that $Val[X/T](Y) = T$ if $Y = X$ else $Val[X/T](Y) = Val(Y)$. The satisfaction's relation $\models_{\mathcal{P}} \subseteq P \times \mathcal{R}eg \times ERL$ is defined as follows :

– $(p, r) \models_{\mathcal{P}} tt$, - $(p, r) \not\models_{\mathcal{P}} ff$,

– $(p, r) \models_{\mathcal{P}} X$ iff $(p, r) \in Val(X)$

– $(p, r) \models_{\mathcal{P}} \varphi_1 \vee (resp. \wedge) \varphi_2$ iff $(p, r) \models_{\mathcal{P}} \varphi_1$ or (resp. and) $(p, r) \models_{\mathcal{P}} \varphi_2$.

– $(p, r) \models_{\mathcal{P}} [g, a]\psi$ iff for every $t \in \mathbb{R}_{\geq 0}$, $(p, g_1, a, p') \in \rightarrow_{\mathcal{P}}$ such that $r+t \models g \wedge g_1$, $(p', r') \models \psi$ for all $r' \in \mathtt{reset}((r + t) \wedge (g \wedge g_1), \{h(a)\})$.

– $(p, r) \models_{\mathcal{P}} \langle g, a\rangle\psi$ iff there exist $t \in \mathbb{R}_{\geq 0}$, $(p, g_1, a, p') \in \rightarrow_{\mathcal{P}}$ such that $r + t \models g \wedge g_1$ and $(p', r') \models \psi$ with $r' \in \mathtt{reset}((r + t) \wedge (g \wedge g_1), \{h(a)\})$.

– $(p, r) \models_{\mathcal{P}} \mu X.\varphi(X)$ iff $(p, r) \in \cap\{T \subseteq S \,|\, \|\varphi(X)\|_{\mathcal{P}}^{Val[X/T]} \subseteq T\}$.

– $(p, r) \models_{\mathcal{P}} \nu X.\varphi(X)$ iff $(p, r) \in \cup\{T \subseteq S \,|\, T \subseteq \|\varphi(X)\|_{\mathcal{P}}^{Val[X/T]}\}$

$\mathcal{P}$ is a model of a formula φ in a time-context r^0 iff $(p^0, r^0) \models_{\mathcal{P}} \varphi$.

The modal operators ($\langle\rangle$ or $[]$) and logic operations ($\wedge$ and $\vee$) are monotonic over the finite set of configurations. The fix-point formulas can be computed using the Knaster and Tarski[10] as follows:

– $(p, r) \models_{\mathcal{P}} \mu X.\varphi(X)$ iff $(p, r) \models \bigcup_{\lambda} \mu^{\lambda} X.\varphi(X)$

– $(p, r) \models_{\mathcal{P}} \nu X.\varphi(X)$ iff $(p, r) \models \bigcap_{\lambda} \nu^{\lambda} X.\varphi(X)$

λ ranges over the class of ordinals.

3. Local model-checking game

3.1. Definition

Given an ERTTS $\mathcal{P}$, an initial time-context r^0, the model-checking game of a formula φ^0 is the tuple $\mathcal{G}(\varphi^0, \mathcal{P}, r^0) = (\mathtt{Pos}, \rightarrow, \mathcal{A}cc)$ where $\mathtt{Pos} \subseteq P \times \mathcal{R}eg \times \mathcal{C}l(\varphi^0)$ are the positions of the game. `Eve`'s positions are those which contain a formula of the form $\langle g, a\rangle\varphi$, $\mu X.\varphi(X)$, $\varphi_1 \vee \varphi_2$. `Adam`'s positions contain a formula of the form $[g, a]\varphi$, $\nu X.\varphi(X)$, $\varphi_1 \wedge \varphi_2$. $\mathcal{A}cc$ is the winning condition on the plays. $\rightarrow \subseteq \mathtt{Pos} \times \mathtt{Pos}$ is the move relation of the game defined as follows:

– There is no move from (ff, p, r) or (tt, p, r).

– There is a move from $(\langle g, a\rangle\varphi, p, r)$ or $([g, a]\varphi, p, r)$ to (resp. φ, p', r') if there is a $t \in \mathbb{R}_{\geq 0}$, $(p, g_1, a, p') \in \rightarrow_{\mathcal{P}}$ such that $r + t \models g_1 \wedge g$ and $r' \in \mathtt{reset}((r + t) \wedge (g \wedge$

$g_1), \{h(a)\})$, else there is a move to (ff, p, r) (resp.(tt, p, r))

– There is a move from $(\varphi_1 \wedge \varphi_2, p, r)$ or $(\varphi_1 \vee \varphi_2, p, r)$ to (φ_j, p, r) with $j \in \{1, 2\}$.

– There is a move from $(\sigma X.\varphi(X), p, r)$ to (U, p, r) where $U = \sigma X.\varphi(X) \in \mathcal{D}(\varphi^0)$.

– There is a move from (U, p, r) to $(\varphi(U/X), p, r)$ if $U = \sigma X.\varphi(X)$ is in $\mathcal{D}(\varphi^0)$

A play in the game $\mathtt{play} = \mathtt{pl}_0.\mathtt{pl}_1.\ldots.\mathtt{pl}_n$ is a finite or infinite sequence of positions such that $\mathtt{pl}_0 = (\varphi^0, p^0, r^0)$. A binding constant U is regenerated in a play if for some i, the formula at the position i is $\psi_i = U$ and the formula at the position $i+1$ is $\psi_{i+1} = \varphi(U)$.

A play is winning for Eve (resp. Adam) if it is finite and the formula at the last position is tt (resp *ff*); or, it is infinite and the oldest binding constant which is regenerated infinitely often is a nu-constant (resp mu-constant). We recall that the order on the constants is total.

A positional strategy for the player j is a mapping $\mathtt{strat}_j : \mathtt{Pos}_j \mapsto \mathtt{Pos}$ where $\mathtt{Pos}_j$ is the set of positions of the player $j \in \{\mathtt{Eve}, \mathtt{Adam}\}$. A play $\mathtt{play} = \mathtt{pl}_0.\mathtt{pl}_1.\ldots.\mathtt{pl}_n$ is consistent with $\mathtt{strat}_j$ if for each $\mathtt{pl}_i$ appearing in the the play, $\mathtt{pl}_i \in \mathtt{Pos}_j$ implies $\mathtt{pl}_{i+1} = \mathtt{strat}_j(\mathtt{pl}_i)$.

A strategy $\mathtt{strat}_{\mathtt{Eve}}$ is a winning strategy if all the plays consistent with it are winning for Eve.

3.2. Game characterisation

Theorem 3.1 *$\mathcal{P}$ is a model of φ^0 in the context-time r^0 iff there is a winning strategy for* Eve *in $\mathcal{G}(\varphi^0, \mathcal{P}, r^0)$.*

We detail in the following tabular the fix-point semantics of the subsection 2.4:

The least fix-point semantics	The greatest fix-point semantics
* $(p, r) \models_{\mathcal{P}} \mu X.\varphi(X)$ iff $(p, r) \models_{\mathcal{P}} \bigcup_\lambda \mu^\lambda X.\varphi(X)$	* $(p, r) \models_{\mathcal{P}} \nu X.\varphi(X)$ iff $(p, r) \models_{\mathcal{P}} \bigcap_\lambda \nu^\lambda X.\varphi(X)$
* $(p, r) \not\models_{\mathcal{P}} \mu^0 X.\varphi(X)$	* $(p, r) \models_{\mathcal{P}} \nu^0 X.\varphi(X)$
* $(p, r) \models_{\mathcal{P}} \mu^{\alpha+1} X.\varphi(X)$ iff $(p, r) \models \varphi(U^\alpha)$ where $U^\alpha = \mu^\alpha X.\varphi(X)$	* $(p, r) \models_{\mathcal{P}} \nu^{\alpha+1} X.\varphi(X)$ iff $(p, r) \models_{\mathcal{P}} \varphi(V^\alpha)$ where $V^\alpha = \nu^\alpha X.\varphi(X)$
* if λ is a limit ordinal, $(p, r) \models_{\mathcal{P}} \mu^\lambda X.\varphi(X)$ iff there is an ordinal $\alpha < \lambda$ such that $(p, r) \models_{\mathcal{P}} \mu^\alpha X.\varphi(X)$.	* if λ is a limit ordinal,$(p, r) \models_{\mathcal{P}} \nu^\lambda X.\varphi(X)$ iff for all the ordinal $\alpha < \lambda$, $(p, r) \models_{\mathcal{P}} \nu^\alpha X.\varphi(X)$.

Following the semantics of the fix-point, we define a signature $\mathtt{sig} = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ as a sequence of ordinals which values depends on a configuration. Let a formula ψ without free variables, a definition list $\mathcal{D}$ containing all the definition constants occurring in ψ, and a configuration (p, r) of $\mathcal{P}$ such that $(p, r) \models_{\mathcal{P}} \mathtt{exp}(\psi)_{\mathcal{D}}$. Let $\mathcal{D}_\mu = (U_{k_1}, \ldots, U_{k_{d}\mu})$ be the projection of $\mathcal{D}$ on μ-constants. ψ has the signature $\mathtt{sig} = (\alpha_1, \ldots \alpha_{d^\mu})$ at the configuration (p, r) if $(\alpha_1, \ldots \alpha_{d^\mu})$ is the least (in lexicographical order) sequence of ordinals such that $(p,r) \models_{\mathcal{P}} \mathtt{exp}_{\mathcal{D}'}(\psi)$ where $\mathcal{D}'$ is a definition list obtained from $\mathcal{D}$ by replacing the $i-th$ definition of the μ-constant $(U_{k_i} = \mu X.\varphi_{k_i}(X)) \in \mathcal{D}$ by $(U_{k_i}^{\alpha_i} = \mu^{\alpha_i} X.\varphi_{k_i}(X)) \in \mathcal{D}'$ for each $i \in \{1,\ldots,d^\mu\}$.

Lemma 3.1 *The signature* $\mathtt{sig}(\varphi,p,r)$ *of* φ *at* (p,r) *is such that:*

– $\mathtt{sig}(\varphi_1 \wedge \varphi_2,p,r) = \max(\mathtt{sig}(\varphi_1,p,r),\mathtt{sig}(\varphi_2,p,r))$

– $\mathtt{sig}(\varphi_1 \vee \varphi_2,p,r) = \mathtt{sig}(\varphi_1,p,r)$ *or* $\mathtt{sig}(\varphi_1 \vee \varphi_2,p,r) = \mathtt{sig}(\varphi_2,p,r)$

– $\mathtt{sig}(\langle g,a\rangle\varphi,p,r) = \mathtt{sig}(\varphi,p',r')$ *for some* (φ,p',r') *such that there is a move from* $(\langle g,a\rangle\varphi,p,r)$ *to* (φ,p',r').

– $\mathtt{sig}([g,a]\varphi,p,r) = sup\{\mathtt{sig}(\varphi,p',r')$ *such that there is a move from* $(\langle g,a\rangle\varphi,p,r)$ *to* $(\varphi,p',r')\}$.

– $\mathtt{sig}(\sigma X.\varphi(X),p,r) = \mathtt{sig}(U,p,r)$ *where* $U = \sigma X.\varphi(X)$ *with* $\sigma \in \{\mu,\nu\}$

– $\mathtt{sig}(U,p,r)$ *with* $U = \mu X.\varphi(X)$ *is greater or equal to* $\mathtt{sig}(\varphi(U/X),p,r)$

– $\mathtt{sig}(V,p,r)$ *with* $V = \nu X.\varphi(X)$ *equal to* $\mathtt{sig}(\varphi(V/X),p,r)$

Proof: Let $\mathcal{D} = (W_1 = \sigma X.\varphi_1(X),\ldots,W_n = \sigma X.\varphi_n(X))$ be the definition list of φ^0. Suppose that $W_i = \mu X.\varphi_i(X)$, $(p,r) \models_{\mathcal{P}} \mathtt{exp}_{\mathcal{D}}(W_i)$ and $\mathtt{sig}(W_i,p,r) = (\alpha_1,\ldots,\alpha_{i-1},\alpha_i,\alpha_{i+1},\ldots,\alpha_{d^\mu})$. Let $\mathcal{D}'$ be a definition list obtained by replacing $(W_i = \mu X.\varphi_k(X))$ by $(W_i^{\alpha_i} = \mu^{\alpha_k} X.\varphi_k(X))$. Let $\psi(X) = \mathtt{exp}_{\mathcal{D}'}(\varphi_i(X))$. It follows from the definition of the signature that $(p,r) \models_{\mathcal{P}} \mu^{\alpha_k} X.\psi(X)$. Since α_i should be a successor ordinal, it follows that $(p,r) \models_{\mathcal{P}} \psi(\mu^{\alpha_i - 1} X.\psi(X))$, which means that the signature of $\psi(\mu^{\alpha_i-1}X.\psi(X))$ at (p,r) is $(\alpha_1,\ldots\alpha_{i-1},\alpha_i - 1,\alpha'_{i+1},\ldots,\alpha'_{d^\mu})$ and is lower that $\mathtt{sig}(W_i,p,r)$. ■

Proposition 3.1 *If* $\mathcal{P}$ *is a model of* φ^0 *in the time-context* r^0 *then, there is a winning strategy for* Eve *in* $\mathcal{G}(\varphi^0,\mathcal{P},r^0)$.

Proof: If $\mathcal{P}$ is a model of φ^0 in the time-context r^0, then $(\varphi^0,p^0,r^0) \models_{\mathcal{P}} \mathtt{exp}_{\mathcal{D}}(\varphi^0)$. Semantically, there is a smallest sequence of ordinals $(\alpha_1,\ldots,\alpha_{d^\mu})$ such that $(\varphi^0,p^0,r^0) \models_{\mathcal{P}} \mathtt{exp}_{\mathcal{D}'}(\varphi^0)$ where $\mathcal{D}'$ is a definition list obtained from $\mathcal{D}$ by replacing the i^{th} definition of the μ-constant $(U_{k_i} = \mu X.\varphi_{k_i}(X)) \in \mathcal{D}$ by $(U_{k_i}^{\alpha_i} = \mu^{\alpha_i} X.\varphi_{k_i}(X)) \in \mathcal{D}^\alpha$ for each $i \in \{1,\ldots,d^\mu\}$.
A strategy which consists to choose at each existential or disjunctive node a successor with the smallest signature is winning because of the considered order on signature. ■

Proposition 3.2 *If there is a winning strategy for* Eve *in* $\mathcal{G}(\varphi^0,\mathcal{P},r^0)$ *then* $\mathcal{P}$ *is a model of* φ^0 *in the time-context* r^0.

Proof: The proof is dual the the above one. One can easily define the signature for ν constants when a configuration (p,r) do not satisfy a formula φ and use that construction to get a contradiction with the existence of a winning strategy for Eve. ■

4. Discussion and Conclusion

We have proposed a game theoretic approach to solve the model-checking problem for ERL formulas and we have shown that the model-checking problem for ERL is decidable. That game can be translated into a parity game by using the alternation depth of fix-point operators as described in [10]. The winning strategy is computable using techniques in [2] since the abstraction provides a two players parity game on a finite arena.
This work provides a theoretical tool which will help us to solve the controller synthesis problem [7] with respect to ERL logic. It also reviews fundamental techniques of fix-point computation and gives an idea of how a model-checking game and a satisfiability game for ERL under real-time systems modelled by timed automata [3] can be defined.

5. References

[1] SOREA M., "A Decidable Fixpoint Logic for Time-Outs", *13th International Conference on Concurrency Theory (CONCUR)*, Lecture Notes in Computer Science, Brno, Czech Republic, August 2002, Springer Verlag, p. 255–271.

[2] VÖGE J., JURDZIŃSKI M., "A Discrete Strategy Improvement Algorithm for Solving Parity Games (Extended Abstract)", EMERSON E. A., SISTLA A. P., Eds., *Computer Aided Verification, 12th International Conference, CAV 2000, Proceedings*, vol. 1855 of *Lecture Notes in Computer Science*, Chicago, IL, USA, July 2000, Springer-Verlag, p. 202–215.

[3] ALUR R. AND, DAVID L. DILL, "A theory of timed automata", *Theoretical Computer Science*, vol. 126(2), 1994, p. 183-235.

[4] STREETT R. S., EMERSON E. A., "An automata theoretic decision procedure for the propositional mu-calculus", *Inf. Comput.*, vol. 81, num. 3, 1989, p. 249–264, Academic Press, Inc.

[5] ALUR R., FIX L., HENZINGER T. A., "Event-Clock Automata: A Determinizable Class of Timed Automata.", *Theor. Comput. Sci.*, vol. 211, num. 1-2, 1999, p. 253-273.

[6] NIWINSKI D., WALUKIEWICZ I., "Games for the mu-Calculus.", *Theor. Comput. Sci.*, vol. 163, num. 1&2, 1996, p. 99-116.

[7] ARNOLD A., VINCENT A., WALUKIEWICZ I., "Games for synthesis of controllers with partial observation.", *Theor. Comput. Sci.*, vol. 1, num. 303, 2003, p. 7-34.

[8] MÜLLER-OLM M., SCHMIDT D. A., STEFFEN B., "Model-Checking: A Tutorial Introduction", *SAS '99: Proceedings of the 6th International Symposium on Static Analysis*, London, UK, 1999, Springer-Verlag, p. 330–354.

[9] KOZEN D., "Results on the Propositional -Calculus.", *ICALP*, 1982, p. 348-359.

[10] ARNOLD A., NIWIŃSKI D., *Rudiments of μ-calculus*, vol. 146 of *Studies in Logic and the Foundations of Mathematics*, North-Holland, 2001.