Image segmentation:

a new watershed transformation algorithm

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RÉSUMÉ. Le but de ce travail est de présenter une nouvelle technique de filtrage et de segmentation d'images en utilisant la théorie de la morphologie mathématique. La technique utilisée est basée sur la ligne des partage des eaux. Afin d'éviter une sur-segmentation, on se propose d'adapter une méthode utilisant le gradient topologique. L'outil Ligne de Partage des Eaux combiné avec un algorithme rapide basé sur l'approche du gradient topologique donne des résultats prometteurs. Les tests numériques proposés montrent l'efficacité de notre méthode pour le problème de la segmentation.

ABSTRACT. The goal of this work is to present a new method for filtering and segmenting an image using mathematical morphology theory. The approach used is based on the watershed transformation. In order to avoid an oversegmentation, we propose to adapt the topological gradient method. The watershed transformation combined with a fast algorithm based on the topological gradient approach gives good results. The numerical tests obtained illustrate the capabilities of our approach in image segmentation.

MOTS-CLÉS : Morphologie mathématique; opérateurs morphologiques; ligne de partage des eaux; gradient topologique; développement asymptotique topologique; segmentation d'images.

KEYWORDS : Mathematical morphology; morphological operators; watershed transformation; topological gradient; topological asymptotic expansion; image segmentation.

1. Introduction

Segmentation is one of the most important problem in image processing. It consists of constructing a symbolic representation of the image : the image is described as homogeneous areas according to one or some attributes chosen a priori. In the literature, we can find various ways of segmentation, the first method is appeared during the sixties and different ways have constantly advanced since the time. The purpose of this work is to adapt a new method for segmenting an image using the watershed transformation [11] and topological asymptotic expansion [7]. In this work we deal with two approaches :

– the topological gradient approach : the goal of topological optimization is to find the optimal decomposition of a given domain in two parts : the optimal design and its complementary. Similarly in image processing, the goal is to split an image in several parts, in particular, in image restoration the detection of edges makes this operation straightforward. The authors in [5] show that it is possible to solve the image restoration problem using topological optimization tools. The basic idea was based on the topological gradient approach used for crack detection [1], in fact an image can be viewed as a piecewise smooth function and edges can be considered as a set of singularities. To solve restoration problem, diffusive methods were associated to the topological gradient to detect edges. Then, by using the same idea, the authors in [2] extend the topological gradient approach to classification problem.

– the watershed transformation is one of the oldest segmentation techniques which was initially due to Beucher and Lantuejoul [3, 4], this technique is well known to be a very powerful segmentation tool. The principle is to consider the image as a topographical relief. This relief is flooded from its minima, when two lakes merge, a dam is built : the set of all dams define the so-called watershed. Efficient algorithms for computing watersheds are described in [3, 11]. One of the advantages of the watershed computation is that it provides always closed contours corresponding to the high crests of the gradient image, which is very usful in image segmentation. However, usually we observe an oversegmentation if one watershed algorithm is applied directly on the image or the gradient image without any treatment. To avoid this oversegmentation, various ways have been proposed in the literature, we can cite for example the techniques based on the markers [13], region merging [12] or scale space approaches [14].
The goal of this work is to deal with the oversegmentation by proposing a new method to solve segmentation problem by combining a fast algorithm using topological gradient approach with a watershed algorithm. The structure of this work is the following. We review in section 2 the topological gradient approach for image restoration and edge detection. According to [8, 9, 10] some principle notions and operators of topological morphology are described in section 3. Numerical tests are presented and discussed in section 4.

2. Application of the topological asymptotic expansion for edge detection

In this section, we use the topological gradient as a tool for detecting edges for image restoration. First, we recall the principle of the topological asymptotic expansion [7] and [1].
Let Ω be an open bounded domain of $\mathbb{R}^2$ and $j(\Omega) = J(u_\Omega)$ be a cost function to be

minimized, where u_Ω is the solution to a given PDE problem defined in Ω. For a small $\rho \geq 0$, let $\Omega_\rho = \Omega \backslash \sigma_\rho$ the perturbed domain by the insertion of a crack $\sigma_\rho = x_0 + \rho\sigma(n)$, where $x_0 \in \Omega$, $\sigma(n)$ is a straight crack, and n a unit vector normal to the crack. The topological sensitivity theory provides an asymptotic expansion of j when ρ tends to zero. It takes the general form

$$j(\Omega_\rho) - j(\Omega) = f(\rho)G(x_0, n) + \circ(f(\rho)), \qquad [1]$$

where $f(\rho)$ is an explicit positive function going to zero with ρ and $G(x_0, n)$ is called the topological gradient at point x_0.
For v a given function in $L^2(\Omega)$, we consider the following problem : find $u_\rho \in H^1(\Omega_\rho)$ such that

$$\begin{cases} -div\,(c\nabla u_\rho) + u_\rho = v & \text{in} \quad \Omega_\rho, \\ \partial_n u_\rho = 0 & \text{on} \quad \partial\Omega_\rho, \end{cases} \qquad [2]$$

where n denotes the outward unit normal to $\partial\Omega_\rho$ and c is a constant function. Edge detection is equivalent to look for a subdomain of Ω where the energy is small. So our goal is to minimize the energy norm outside edges

$$j(\rho) = J(u_\rho) = \int_{\Omega_\rho} \|\nabla u_\rho\|^2. \qquad [3]$$

By considering p the solution to the adjoint problem

$$\begin{cases} -div(c\nabla p) + p = -\partial_u J(u) & \text{in} \quad \Omega, \\ \partial_n p = 0 & \text{on} \quad \partial\Omega, \end{cases} \qquad [4]$$

we obtain in the case of a crack $\sigma_\rho(n)$ with boundary condition $\partial_n u = 0$ on $\partial\sigma_\rho(n)$, the following topological asymptotic expansion.

$$j(\rho) - j(0) = \rho^2 G(x_0, n) + \circ(\rho^2), \qquad [5]$$

with

$$G(x_0, n) = -\pi c(\nabla u(x_0).n)(\nabla p(x_0).n) - \pi|\nabla u(x_0).n|^2. \qquad [6]$$

The topological gradient could be written as

$$G(x, n) =< M(x)n, n >, \qquad [7]$$

where $M(x)$ is the symmetric matrix defined by

$$M(x) = -\pi c \frac{\nabla u(x)\nabla p(x)^T + \nabla p(x)\nabla u(x)^T}{2} - \pi\nabla u(x)\nabla u(x)^T. \qquad [8]$$

For a given x, $G(x, n)$ takes its minimal value when n is the eigenvector associated to the lowest eigenvalue λ_{min} of M. This value will be considered as the topological gradient associated to the optimal orientation of the crack $\sigma_\rho(n)$.

3. An overview of mathematical morphology

One of the aim of this work is to show how the use of mathematical morphology operators can be very useful in image segmentation. We describe briefly in this section the principle notions and operators we use. Let $u(x,y)$ with $(x,y) \in \mathbb{R}^2$, be a scalar function describing the image. The so-called structuring element is a set $D \subset \mathbb{R}^2$ that determines the neighbourhood relation of pixels with respect to a shape analysis task, this element is usually chosen as convex set. In our numerical tests, we choose the structuring element as a disk.
$u \oplus D$ denotes the flat dilatation of u by the structuring element D and $u \ominus D$ denotes the flat erosion of u by the structuring element D. $u \oplus D$ replaces the grey level value of the image u by its maximum within a mask defined by D and $u \ominus D$ is determined by taking the minimum

$$(u \oplus D)(x,y) = Sup\left\{f(x-\acute{x}, y-\acute{y}), (\acute{x},\acute{y}) \in D\right\}, \qquad [9]$$

and

$$(u \ominus D)(x,y) = Inf\left\{f(x+\acute{x}, y+\acute{y}), (\acute{x},\acute{y}) \in D\right\}. \qquad [10]$$

The opening operation, denoted by $\circ$, is defined as erosion followed by dilatation

$$u \circ D = (u \ominus D) \oplus D, \qquad [11]$$

and the closing operation which is denoted $\bullet$, is consisting of a dilatation followed by an erosion

$$u \bullet D = (u \oplus D) \ominus D. \qquad [12]$$

We define the White Top Hat as the difference between the original image and its opening

$$WTH(u) = u - (u \circ D), \qquad [13]$$

and its dual gives the Black Top Hat, which is the difference between the closing and the original image

$$BTH(u) = (u \bullet D) - u. \qquad [14]$$

In the field of mathematical morphology, to define the contours of an image, Beucher [9] introduce a morphological gradient, which is also called the Beucher gradient and given by

$$\delta_D u = (u \oplus D) - (u \ominus D), \qquad [15]$$

and the morphological Laplacian is given by

$$\Delta_D u = (u \oplus D) - 2u + (u \ominus D). \qquad [16]$$

We note here that this morphological Laplacian allows us to distinguish between influence zones of minima and suprema : regions with $\Delta_D u < 0$ are considered as influence zones of suprema, while regions with $\Delta_D u > 0$ are influence zones of minima. Then $\Delta_D u = 0$ allows us to interprete edge locations, and will represent an essential property for the construction of morphological filters. The basic idea is to apply either a dilatation or an erosion to the image u, depending on whether the pixel is located within the influence zone of a minimum or a maximum. Many variants of morphological filters ca be found in the literature. In our numerical tests, we have considered the Alternating Sequential Filters ASF that consist to take an original noisy image v, and first to filter it by a small closing ϕ_1 followed by a small opening γ_1, then by a slightly larger closing ϕ_2, followed by a slightly larger opening γ_2, etc. The final operator produced by this succession of openings and closings gives the notion of the ASF. For a more detailed treatment of this topic the reader is referred to [10].

4. Numerical Applications

1) First, we consider the problem of denoising an image and preserving features such as edges. According to section 2, the idea of the topological gradient approach consists in inserting small cracks in regions where the topological gradient λ_{min} is smaller than a negative given threshold. These regions represent the edges of the image. Note that from a numerical point of view, it is more convenient to simulate cracks by a small value of c. The numerical results of this approach are illustrated in Figure 1. A Gaussian noise ($\sigma = 20$) is added to the original image which grey level take its values in the interval $[0, 255]$. We note that the topological gradient algorithm requires 3 systems resolution to obtain the reconstruction of the original image u.

2) Second, we present numerical tests for the segmentation problem using mathematical morphology tools. More precisely, the approach used in this work is based on the watershed technique by considering an algorithm proposed by P. Soille [11]. Figure 2 illustrates this technique. To avoid the problem of oversegmentation seen in the Figure 2 , we propose to filter the image by using a ASF algorithm, this technique is very common in the literature. We obtain 449 regions in the first case (image without treatment) while the image treated before segmentation gives 273 regions in the case of 2 iterations applied on the ASF algorithm and 187 regions if 3 iterations are applied on the ASF algorithm. It should be noted that it is not interesting to run any more the ASF algorithm because of the loss of the edges. Finally, we note that the segmentation sheme of a 131^2 image on a PC Pentium4, 512 Mo DDR, requires around 100 CPU seconds. ($73s$ for the first step and respectively $125s$ and $101s$ for the second and the third step).

3) Third, we propose a new algorithm for the segmentation problem which combine the topological gradient approach with the LPE technique. As the topological gradient approach is well-suited for edge preserving image denoising such that the image is restored at the first iteration of the optimization process, and the computational cost of this iteration can be reduced drastically using spectral methods [6], then we propose to avoid the oversegmentation seen before, by using a simple topological strategy. In our case, we consider a modification of the equation 2 as follows

$$\begin{cases} -div\,(c(\epsilon)\nabla u_\rho) + u_\rho = v & \text{in} \quad \Omega_\rho, \\ \partial_n u_\rho = 0 & \text{on} \quad \partial\Omega_\rho, \end{cases} \qquad [17]$$

with $c(\epsilon) = \epsilon$ on the edges and $\frac{1}{\epsilon}$ elsewhere, with ϵ a positive constant tending to zero. This modification is a particular example where smoothing on both sides of an edge is much stronger then smoothing across it. Our new algorithm is then based on the precedent restoration algorithm according to equation 17. Then, by considering the topological gradient determined after the restoration process, we simply apply the LPE algorithm used in step 2 of numerical applications. The topological gradient replaces in our new algorithm the morphological gradient classically used in LPE algorithms. The numerical results of this new approach are illustrated in Figure 3. It should be noted that we obtain 157 homogeneous regions and our new algorithm requires around 370 CPU seconds.

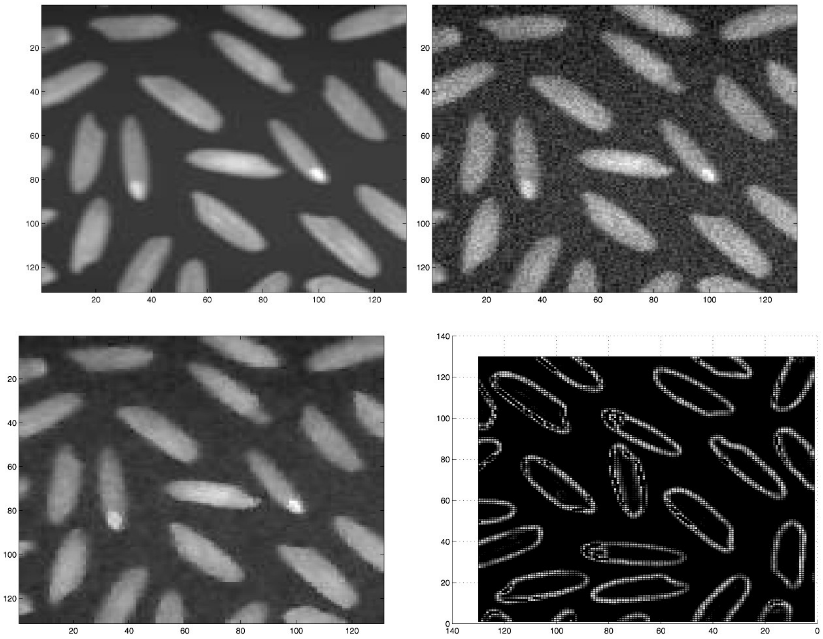

Figure 1. *top left : original image, top right : noisy image* $\sigma = 20$*, down left : restored image by topological gradient approach, down right : edges of the image restored*

5. Conclusion

We have presented in this work a new approach for the segmentation problem taking advantage of the topological gradient approach and a simple watershed algorithm. The numerical results obtained are very promising. The algorithm proposed is efficient since any treatment or a priori information on the image was taking account. However, the authors would like to improve the numerical results presented in this work, as computational time, by considering other watershed algorithms tested in the literature.

6. Bibliographie

[1] S. AMSTUTZ, I. HORCHANI, M. MASMOUDI, « Crack detection by the topological gradient method », *Control and Cybernetics*, n° 34, 2005.

[2] D. AUROUX, L. JAAFAR BELAID, M. MASMOUDI, « A topological asymptotic analysis for the regularized grey-level image classification problem », *Mathematical Modelling and Numerical Analysis*, n° 41, 2007.

[3] S. BEUCHER, »Segmentation d'image et Morphologie mathématique », *Thèse de Doctorat, Ecole Nationale Supérieure des Mines de Paris, juin 1990.*

[4] S. BEUCHER, C. LANTUEJOUL, « Use of Watersheds in Contour Detection », *Proc. Int. Workshop on image processing Rennes, France, 1990.*

[5] L. JAAFAR BELAID, M. JAOUA, M. MASMOUDI, M.L. SIALA, « Image restoration and edge detection by topological asymptotic expansion », *CRAS*, n° 342, 2006.

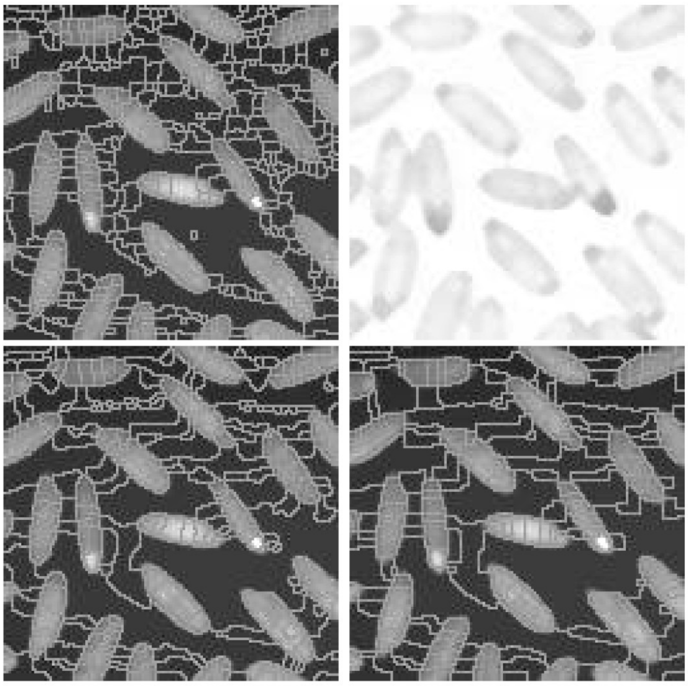

Figure 2. *top left : first step of segmentation, top right : filtered image using 2 iterations, down left : second step of segmentation (2 iterations of the ASF algorithm), down right : third step of segmentation (3 iterations of the ASF algorithm)*

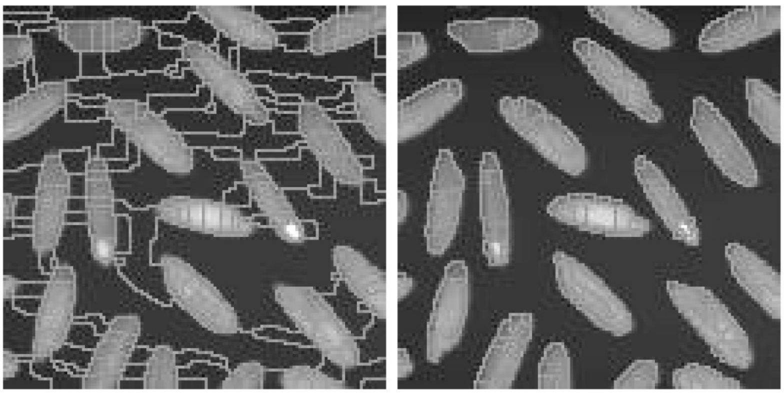

Figure 3. *left : image segmented by classical LPE algorithm with 3 iterations of the FAS algorithm, right : image segmented using an LPE algorithm applied to topological gradient*

[6] L. JAAFAR BELAID, M. JAOUA, M. MASMOUDI, M.L. SIALA, « Application of the topological gradient to image restoration and edge detection, », *J. of Boundary Element Methods*, n° to appear.

[7] M. MASMOUDI, , « The Topological Asymptotic », *Computanional Methods for Control Applications, R. Glowinski, H. Kawarada and J. Périaux Eds, International Series GAKUTO, Tokyo, Japan*, n° 16, 2001.

[8] G. MATHERON, J. SERRA, « The birth of mathematical morphology », *Ecole des Mines de Paris - Centre de Morphologie Mathématique, 1998*.

[9] J. SERRA, « Image Analysis and Mathematical Morphology », *Academic Press, London*, n° I, 1982.

[10] J. SERRA, « Image Analysis and Mathematical Morphology : Theoretical Advences », *Academic Press, London*, n° II, 1988.

[11] P. SOILLE, « Morphologie mathématique : Du relief à la dimensionnalité -Algorithmes et méthodes- », *Thèse de Doctorat, La Faculté des Sciences Agronomiques de l'Université Catholique de Louvrain, 1992.*

[12] L. VINCENT, P. SOILLE, « Watersheds in digital spaces : an efficient algorithm based on immersion simulations », *IEEE trans. Patt. Anal Mach. Intelle*, n° 13, 1991.

[13] F. MEYER, S. BEUCHER, « Morphological segmentation », *J of Visual communication and Image representation*, n° 1, 1990.

[14] P.T. JACKWAY, « Gradient watersheds in morphological scale space », *IEEE trans. Image Processing*, n° 5, 1999.