Non-Rigid Multimodal Registration Using Finite Element Method and Mutual Information - Theory

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RÉSUMÉ. Le but est de pouvoir faire un recalage beaucoup plus rapide. Cette méthode peut être implémentée sur un large domaine d'application notamment en médecine, en paléontologie et en télédétection. On peut considérer le recalage comme la résolution d'un problème de mise en correspondance et qu'on cherche les déformations qu'on doit appliquer. On a modélisé les images comme des matériaux élastiques linéaires isotropes. Aussi, cette modélisation permet de donner les caractéristiques de régularisation de l'image. La combinaison de la régularité du matériau et de l'énergie de similarité nous a permis de faire cette étude de recalage multimodal non-rigide. On a fait une implémentation en 2D pour vérifier la validité l'étude du processus.

ABSTRACT. The aim is to do a non-rigid registration with a fast algorithm. A large domain of application is involved like medicine, palaeontology and remote sensing. The registration is the process of solving the correspondence between two images. In fact, the objective is to find the deformation to apply to an image while referring to the other image. So, the images are in isotropic linear elastic material model. This modelling permits to give the regularization characteristics. However, the mutual information describes by Mattes is used like the similarity metric. The combination of the regularization of the material and the energy of similarity allowed us to do this survey. To verify the validity the survey of the process, we have implemented tests in 2D.

MOTS-CLÉS : recalage, modèle élastique, méthode des éléments finis, information mutuelle, imagerie médicale.

KEYWORDS : Registration, elastic model, finite elements method, mutual information, medical imagery.

1. Introduction

Comparing two images of the same patient at different moments or superimposing images from a model and from a patient are daily tasks in medical imagery. Analyzing satellite images of a same region acquired at different moments and/or from different types of sensors is important in remote sensing domain. Comparing images of samples with models is usual work of palaeontologists. Often, these images are not in the same geometric reference mark. They can also be acquired with different imagery techniques. Besides, some local deformations are generated during acquirements. Thus, the main problem is to set in correspondence these images. This process is called registration. In other term, registration is the process of finding transform that maps the points from moving image to corresponding points in the reference image. One can divide registration techniques in two groups : rigid global registration and non-rigid local registration. In rigid global registration, the objective is to find linear transforms that minimize global differences between reference image and moving image. In non-rigid local registration, the aim is to find the local deformations. Our work is restricted in non-rigid local registration. Indeed, we suppose here that both images are already globally registered, but some "small deformations" still subsist. We also suppose that deformations are due either to acquirement setting (positions, parameters, …), either to subject's movements (for example distortion of lungs during inspiration/expiration, …). Both images may possibly be obtained from two different imagery techniques. As we suppose, between two images, the differences consist only of local deformations. Therefore, one can consider two competing energies : similarity (external) energy and regularization (internal) energy. In this paper, we present a new approach of a non-rigid local registration. We have used elastic as material model; we have taken Mattes mutual information [9] as metric and we consider finite elements method for solving. After a brief presentation of non-rigid approach, we will present a new algorithm to find local deformations. We will develop each step and we will present examples with bio-medical data.

2. Non-rigid registration approach

Several studies have already been led about non-rigid registration methods. Thus, we can find several representations models such as the mechanical model, the fluid model [7], the Splines model [2], …. As solving techniques, one can mention the optical flow method [13], the free form deformations [12] …. For all registration model and technique, the objective is to find local deformations that are necessary to apply to an image. There exists a modelling phase permits to determine the image regularization. It is especially based on the solving technique. There is also a procedure of optimization permits to get the transformation's parameters with an objective to converge towards an optimum. The most known and most used are : downhill simplex [11], gradient descent [11], conjugate gradient [11], and quasi-Newton (LBFGS) [10]. As similarity measure, one can find mean squares metric, normalized correlation metric and mutual information metric. Mutual information measures how much information a random variable (image intensity in first image) tells about another one (image intensity in second image) [14], [9], [8]. What motivated us to make this work is that the already led studies have problems. Either, it has a problem of the regularity such as [13] and [12], or the algorithm is too slow and/or asks for exaggerated resources [6], or the approach is not adapted with images in low resolution [2]. Indeed, for example, for free form deformations [12], the displacement of one node affects only some pixels (voxels). However, we know that a registration is not a morphing process. By implementing the optical flow method [13] with Insight Toolkit (ITK), we were able to notice that some parts of the image disappear after registration. So, the exploitation of the elastic model with finite elements methods is more interesting.

3. Proposed algorithm

In our survey, we adopt an isotropic elastic linear model. This model is very efficient when having small deformations. Deformations will be obtained by making an approximation material behaviour with finite elements approach. We also take the maximization of the mutual information, precisely the variant proposed by [9], as optimisation method.

3.1. Elastic model and finite elements method

Let us consider that images are made with isotropic linear elastic material. Therefore, it must respect behaviour of an elastic body. According to the Navier-Stokes model, we have :

$$\mu\nabla^2 u(X) + (\lambda+\mu)\nabla(\nabla u(X)) + f(X) = 0 \qquad \textbf{(1)}$$

where f is external force, is displacement field, and are Lamé's ratio. This last equation is deduced from $\sigma = \lambda tr(\varepsilon)I + 2\mu\varepsilon$ where σ is the constraint tensor at point X, ε is the deformation tensor at X, $tr(\varepsilon) = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$ is trace of matrix ε and I is the identity matrix. We can also be written

$$\varepsilon = \frac{\nu}{E} tr(\sigma)I + \frac{1+\nu}{E}\sigma \qquad \textbf{(2)}$$

with E is Young modulus and ν Poisson's ratio. The objective is to solve partial derivatives equation (1). We calculate the displacement $u(X)$ at point X when a force $f(X)$ is applied. Solving technique using finite elements method is used to find an approached solution of this problem. Indeed, it consists in dividing an image in several elements named finite elements and solving the problem by pieces.

3.2. Mutual information

Mutual information is a tool from information theory. It measures the quantity of information from an image gives on another one. Expression of mutual information is given by $MI(F,M) = H(F) + H(M) - H(F,M)$ with F the reference image, M the moving image and $H(X)$ is entropy of X

$$H(X) = -\int p_X \log p_X dx \text{ and } H(X,Y) = -\int\int p_{xy} \log p_{xy} dxdy \qquad \textbf{(3)}$$

To obtain probabilities distribution, two methods are generally used : a method based on a discrete approach [3] ,[8] and a method with evaluation of probability density functions. The estimated probability density is expressed as :

$$p(x) \approx p^*(x) = \frac{1}{N_A} \sum_{x_i \in A} K(x - x_i) \qquad \textbf{(4)}$$

With A : sample, N_A : size of sample, x_i : values of sample, K : kernel function (Gauss ou B-spline, …)

However, several variants of the mutual information computation using the estimated probability density function also exist, such as a method based on empiric average entropy computation [14]; an approach proposed by [9], a method based on the calculation of normalized quadratic mutual information [1],… We choose here variant proposed by [9]. It consists of dividing joint histogram into regular bins and to evaluate the estimated probability p^* on every bin. The principle as follows : with a sample, we calculate the joint histogram $p^*(k,l)$. Then we calculate the marginal probabilities $p^*_m(k)$ (for the moving image) and $p^*_f(l)$ (for reference image). For the detailed calculations, refer to [9]. This variant according to [9] has two main advantages : the calculations of density probabilities are fast and the objective function is smooth.

3.3 Method

Combination of hypothesis about an elastic material model, the finite elements method solving and the Mattes mutual information metric is very attractive to compute non-rigid registration.

3.3.1 Problem solving

We opted for : isotropic linear elastic model as representation model, finite element method as solving method of the registration problem and maximization of the mutual information with the variant proposed by [9] as optimization technique. Indeed, we consider all objects of images made with a same isotropic linear elastic material. Then, one can construct mesh to divide images in elements. Rigidity matrix K will be obtained from mesh and elastic parameters of the images (Poisson's ratio and Young modulus). Finally, one uses the mutual information as similarity metric to compute the subject external strengths. We solve the partial derivative equations with the finite elements method while trying to construct a linear system :

$$KU = F \tag{5}$$

where K is the global rigidity matrix, U is the displacement fields and F is the similarity force. Evidently K contains the criteria of regularity of the elastic behaviour, and F is proportional to the gradient of the similarity measure. The mesh chosen in this survey is a uniform grid. This choice is based on its simplicity and because the time of construction is faster than for an adaptive grid.

3.3.2. Transformation

Let us consider a point X belonging to an element. Transformation of X is a translation relatively to the node displacements of this element. So the Jacobian of the transformation in the element is given by

$$J_e = \begin{pmatrix} N_0(X) & N_1(X) & ... & 0 & 0 & ... & 0 & ... \\ 0 & 0 & ... & N_0(X) & N_1(X) & ... & 0 & ... \end{pmatrix} \tag{6}$$

where $[N(X)]$ is the matrix interpolation of displacements

3.3.3. Deformation, constraint and rigidity matrix K

In a linear isotropic elastic model, relation between deformation and constraint is given by $\sigma = [D_e]\varepsilon$ where $[D_e]$ is the elasticity matrix. For each element, we can calculate the rigidity matrix K_e with

$$[K_e] = \int_{Ve} {}^t[B_e][D_e][B_e]dV \tag{7}$$

where $[B_e]$ is gradient's matrix of $[N(X)]$, and V_e is the volume of the element.

From the K_e matrices, we obtain the matrix of global rigidity K. Let us notice that the rigidity matrix K is symmetric positive definite.

3.3.4. Similarity force F

Nodal forces f_i are proportional to components of the similarity energy gradient. So, we have

$$F = \alpha \nabla S \quad \textbf{(8)}$$

is gradient of mutual information measure and α is a positive coefficient.

3.3.5. Registration and hierarchical approach

The obtained equations system is resolved with conjugate gradient approach according to [5]. We adopted the multi-resolution approach for mesh i.e we start with a coarser mesh toward a thinner mesh. Hierarchical approach permits rapid convergence of minimum research. Therefore, at the end of a registration step, one uses calculated nodes displacements as initial values for the following hierarchy process. Thus, this approach permits to accelerate processing and increase the robustness of the registration algorithm.

```
For each h ∈ hierarchies
   Q_h nodes generation;
   Calculate the u_h from the nodes Q_{h-1} and their displacements u_{h-1} ;
   Construction of K_e from u_h ;
   Assemble K_e to get K and calculate K^{-1} ;
   g = gradient(F, MoT_u) ;
   d = -g ; // The descent direction;
   Repeat
          s = ||g||^2 ;
          τ = K^{-1} d; // τ becomes as a descent direction
          t = Optimal_step(F, MoT_u, τ) ;
          u = u + t * τ ;
          g = gradient(F, MoT_u) ;
          β = ||g||^2 / s ;
          d = -g + βd ; // If thrown back the algorithm, then d = -g
   until ||t * τ|| ≤ epsilon
End for
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Figure 1. *Non-rigid registration algorithm*

4. Test images and results

To experiment this method, we have tested with 2D images. We executed calculations on a low-end PC (1.8 Ghz processor and 384 Mb DDR). The images sets used for this test consist of sample images from MRI and MRI, PET and SPECT. One used about twenty images from different scans like thorax, brain, ... We have considered the images as a combination of viscous liquid and flesh, and we have taken as characteristics : Poisson's ratio ν =0.1 and Young modulus E=0.002. We have done about twenty tests. In without hierarchical approach, for each test, the execution time is about 20 seconds after 40 iterations, and we have about 15% to 35% of gain. Therefore, using four hierarchies, the execution time is about 70 seconds and after 32 iterations, we have about 15% to 55% of gain. We can notice that the multi-resolution approach of the mesh can intervene in the domain of the "large deformations".

5. Conclusion

In our registration method, we used an elastic model, the finite elements method and the maximization of the mutual information according to [9] in a uniform grid. We could notice that the approach using mutual information according to [9] is faster than with other variants. Besides, the objective function is also sufficiently smooth to avoid falling in a local minima. However, we tested with an image intensities-based adaptive grid and we had more interesting result but the mesh construction proves to be very long. With thinner mesh, time of adaptive grid construction is about 50 times more than with uniform grid, with only less than 0.4% of mutual information value improvement. As well, we can observe two advantages. The algorithm is fast and, with elastic model, one has no regularity problem. Finally, we could notice that this registration method gives good results with low-resolutions images as well as that with of high-resolutions images. We will implement this method for large size and high-resolution images : 3D medical images, satellites images and palaeontology images. As registration processes requires an important amount of computation and memory resources. We will also study the portage of these sequential algorithms toward parallel ones.

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