Self Adjoint and m-Accretive Splitting iterative Method for Solving neutron Transport equation in 1-D Spherical Geometry

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ABSTRACT. This paper presents an iterative method based on a Self adjoint and m-Accretive splitting for the numerical treatment of the steady state neutron transport equation in 1-D spherical geometry. Theoretical results show the convergence of the method. The convergence of the method is numerically illustrated and compare with the standard source iteration method on a sample problem.

RÉSUMÉ. Ce papier présente une methode iterative basée sur une décomposition d'opérateur pour le traitement numérique de l'équation du transport des neutrons en géométrie sphérique 1-D. Les résultats de convergence théorique de la méthode sont présentés. La convergence de cette méthode est numériquement illustrée sur un exemple et comparée à la méthode standard des sources d'itération

KEYWORDS : Transport equation, spherical geometry, self adjoint operator, m-accretive, operator splitting, iterative methods.

MOTS-CLÉS : Equation du transport, géometrie spherique, décomposition d'opérateurs, opérateur auto-adjoint,opérateur m-accretif, méthodes itératives

1. Introduction and mathematical setting

This paper focus on developing iterative methods for the numerical treatment of the single group steady state first order neutron transport equation in 1-D spherical geometry verified by the neutron flux $u(r,\mu)$, solution of:

$$\mu\frac{\partial u}{\partial r}+\frac{1-\mu^2}{r}\frac{\partial u}{\partial \mu}+\sigma u=\int_{-1}^{1}\kappa(r,\mu,\mu')u(r,\mu')d\mu'+q(r,\mu) \text{ in } (0,R)\times(-1,1), \quad (1)$$

where the region occupied by the particles is a sphere of radius $R>0$; r is the distance from the center of the sphere; μ is the cosine of the angle the particle velocity makes with the radius; $\sigma(r)$ is the total cross section; $\kappa(r,\mu,\mu')$ is a positive kernel specifying the scattering of particles; $q(r,\mu)$ is a known particles source and $u(r,\mu)$ represents the angular flux to be determined for all point $r\in(0,R)$ and all $\mu\in(-1,1)$. The boundary conditions prescribing the inflow of particles into the sphere reads:

$$u(R,\mu)=0 \text{ for } \mu\in(-1,0). \quad (2)$$

We introduce a two-step iteration method linked to a Self-adjoint and m-Accretive Splitting (SAS) of the transport operator. We investigate the convergence of this iterative method. Theoretical analysis shows that this iterative method converges unconditionally to the solution of the transport equation. An upper bound of the contraction factor of the iteration is derived. It is dependent on the spectrum of the Self-adjoint part of the transport operator, but is independent of the spectrum of its m-Accretive part as well as the eigenfunctions of the operators involved. The convergence of the method is numerically illustrated and compare with the standard Source Iteration method on a sample problem.

Let $\Omega = (0, R) \times (-1, 1)$. We define the space

$$W^2(\Omega) = \left\{ u \in L^2(\Omega), \mu \frac{\partial u}{\partial r} \in L^2(\Omega) \text{ and } \frac{1-\mu^2}{r} \frac{\partial u}{\partial \mu} \in L^2(\Omega) \right\}, \tag{3}$$

endowed with the norm $\|u\|^2_{W^2} = \|u\|^2 + \left\| \mu \frac{\partial u}{\partial r} + \frac{1-\mu^2}{r} \frac{\partial u}{\partial \mu} \right\|^2$, where $\|.\|$ denotes the standard $L^2(\Omega)$ norm. The functions of $W^2(\Omega)$ have traces on $\{0\}$ and on $\{R\}$ in $L^2(-1, 1)$ [6].

Let A, Σ, K and T be the operators defined in $W^2(\Omega)$ by:

$$Au = \mu \frac{\partial u}{\partial r} + \frac{1-\mu^2}{r} \frac{\partial u}{\partial \mu}, \quad Ku = \int_{-1}^{1} \kappa(r, \mu, \mu') u(r, \mu') d\mu', \quad \Sigma u = \sigma(r) u \tag{4}$$

and

$$Tu(r, \mu) = Au(r, \mu) + \Sigma u(r, \mu) - Ku(r, \mu). \tag{5}$$

In the operator form, the problem (1-2) may be written as

$$Tu(r, \mu) = q(r, \mu). \tag{6}$$

We have

$$\mathcal{D}(K) = \mathcal{D}(\Sigma) = L^2(\Omega). \tag{7}$$

It follows that

$$\mathcal{D}(T) = \mathcal{D}(A) = \left\{ u \in W^2(\Omega); u(R, \mu) = 0 \text{ for } \mu < 0 \right\}. \tag{8}$$

Proceeding as in [1, 2, 3], we make the following assumptions:

$(A1)$ $\sigma \in L^\infty((0, R))$, $\exists \sigma_0 > 0$ such that $\sigma(x) \geq \sigma_0$ a.e. on $(0, R)$.

$(A2)$ $\kappa(r, \mu, \mu') = \kappa(r, \mu', \mu)$ and κ is positive .

$(A3)$ $\exists c \in [0, 1), \int_{-1}^{1} \kappa(r, \mu, \mu') d\Omega' \leq \sigma_0 c$ a.e. on Ω.

The above assumptions guarantee the following results [1, 6, 8] .

Proposition 1.1 *The operators T, K and A verify the following properties:*

1) The operator T is positive definite.

2) The operators K and $\Sigma - K$ are selfadjoint and positive definite.

3) The operator A is m-accretive [6].

It then follows from Proposition 1.1(1) that the solution of problem (1)-(2) exists and is unique in $\mathcal{D}(T)$, and from Proposition 1.1(2 - 3) that the operator T admits a self adjoint and m-accretive splitting.

The paper is organized as follows. In Section 2, we present the two step iterative methods and the convergence analysis. Section 3 deals with an implementation of the method. The discretization and the numerical results are given in Section 4. Some concluding remarks are given in section 5.

2. The Self-adjoint m-Accretive Splitting (SAS) Methods.

The standard splitting of the transport operator consists of a decoupling between its differential and integral parts [6]. This splitting leads to the source iteration method defined by: given $\psi_0 \in \mathcal{D}(T)$, solve

$$\begin{cases} L\psi^{(n+1)} = K\psi^n + q \text{ in } \Omega \\ \psi^{(n+1)} \in \mathcal{D}(T) \end{cases}, \tag{9}$$

where $L = A + \Sigma$. This method becomes extremely slow in the critical case.

Let us consider now another natural splitting of the transport operator stated as follows:

$$-T = -(A + S), \tag{10}$$

where $S = \Sigma - K$. Therefore for any positive constant α, we have the following two-step splitting:

$$T = (\alpha I + S) - (\alpha I - A) = (\alpha I + A) - (\alpha I - S). \tag{11}$$

Since A is m-accretive and S is bounded and self-adjoint in $L^2(\Omega)$, it follows that for $\alpha > 0$, the operator $(\alpha I + A)^{-1}$ is bounded from $L^2(\Omega)$ to $D(T)$ and the operators $(\alpha I + S)^{-1}$ and $\alpha I - S$ are bounded and self-adjoint.

For $\alpha > 0$, the splitting (11) leads to the following two-step iteration method for the solution of the problem (1-2).

Given an initial guess $\psi^{(0)} \in D(T)$, for $k = 0, 1, \ldots$ until $\{\psi^{(k)}\}$ converges, calculate

$$\begin{cases} (\alpha I + S)\psi^{(k+\frac{1}{2})} = (\alpha I - A)\psi^{(k)} + q \\ (\alpha I + A)\psi^{(k+1)} = (\alpha I - S)\psi^{(k+\frac{1}{2})} + q \end{cases}. \tag{12}$$

Therefore, $\psi^{(k+1)}$ satisfies

$$(\alpha I + A)\psi^{(k+1)} = M(\alpha)(\alpha I + A)\psi^{(k)} + N(\alpha)q, \tag{13}$$

where $N(\alpha) = 2\alpha(\alpha I + S)^{-1}$ and

$$M(\alpha) = (\alpha I - S)(\alpha I + S)^{-1}(\alpha I - A)(\alpha I + A)^{-1}. \tag{14}$$

Therefore, the exact solution ψ^* of equation (5) verifies

$$\|\psi^{(k+1)} - \psi^*\|^* \leq \|M(\alpha)\|\|\psi^{(k)} - \psi^*\|^*. \tag{15}$$

Where $\|\psi\|^* = \|(\alpha I + A)\psi\|$ $\psi \in W^2$. Since the application $\|.\|^*$ is a norm in $W^2(\Omega)$, the iterative method (12) converges if

$$\|M(\alpha)\| < 1. \tag{16}$$

Proposition 2.1 *Convergence of the SAS iteration method.*

Let α be a positive constant. The norm $\|M(\alpha)\|$ of the operator $M(\alpha)$ is bounded by

$$\beta(\alpha) = \sup_{\lambda \in \sigma(S)} \left| \frac{\alpha - \lambda}{\alpha + \lambda} \right| < 1, \tag{17}$$

where $\sigma(S)$ is the spectrum of the operator S. Therefore it holds that $\|M(\alpha)\| < 1$ and the SAS iteration converges to the unique solution $\psi^* \in \mathcal{D}(T)$ of the problem (1)-(2). The optimal parameter $\bar{\alpha}$ which minimizes the bound $\beta(\alpha)$ is given by

$$\bar{\alpha} = \sqrt{\lambda_{\min}\lambda_{\max}} \tag{18}$$

and

$$\beta(\bar{\alpha}) = \frac{\sqrt{\lambda_{\max}} - \sqrt{\lambda_{\min}}}{\sqrt{\lambda_{\max}} + \sqrt{\lambda_{\min}}}, \tag{19}$$

where $\lambda_{\min}$ and $\lambda_{\max}$ denote respectively the lower and the upper bounds of the spectrum of the operator S.

Proof. For $\alpha > 0$, let us set $U(\alpha) = (\alpha I - A)(\alpha I + A)^{-1}$ and $V(\alpha) = (\alpha I - S)(\alpha I + S)^{-1}$. $U(\alpha)$ and $V(\alpha)$ are bounded operators. They map $L^2(\Omega)$ into $\mathcal{D}(T)$ and $L^2(\Omega)$ respectively. We have $\|M(\alpha)\| \leq \|U(\alpha)\| \|V(\alpha)\|$. Since S is a bounded self-adjoint operator in $L^2(\Omega)$, we have

$$\|V(\alpha)\| = \|(\alpha I - S)(\alpha I + S)^{-1}\| = \sup_{\lambda \in \sigma(S)} \left| \frac{(\alpha - \lambda)}{(\alpha + \lambda)} \right| = \beta(\alpha)$$

It holds from the positivity of the real α and λ that $\beta(\alpha) < 1$.

The proof of the proposition is achieved if the norm of operator $U(\alpha)$ verifies $\|U(\alpha)\| \leq 1$. Since A is m-accretive, $(\alpha I + A)$ is bijective from $D(A)$ to $L^2(\Omega)$, the operator $(\alpha I + A)^{-1}$ is bounded and for $\psi \in D(T)$, $(A\psi, \psi) \geq 0$. Moreover, we have

$$\|(\alpha I - A)\psi\|^2 - \|(\alpha I + A)\psi\|^2 = -4\alpha(A\psi, \psi) \leq 0.$$

Taking $\psi = (\alpha I + A)^{-1}\varphi, (\varphi \in L^2(\Omega))$, we obtain $\|U(\alpha)\varphi\|^2 \leq \|\varphi\|^2$. It follows that $\|U(\alpha)\| \leq 1$.

To prove (18) and (19), we consider on $\sigma(S)$ the function $t_\alpha(\lambda) = \frac{\alpha - \lambda}{\alpha + \lambda}, \alpha > 0$. t_α is a decreasing function. It follows that $|t_\alpha|$ gets his maximal value at $\lambda_{\min}$ or $\lambda_{\max}$. Thus $\beta(\alpha) = \max\left\{ \left| \frac{\alpha - \lambda_{\min}}{\alpha + \lambda_{\min}} \right|, \left| \frac{\alpha - \lambda_{\max}}{\alpha + \lambda_{\max}} \right| \right\}$. The minimal value of $\beta(\alpha)$ is obtained when [5] $\frac{\alpha - \lambda_{\min}}{\alpha + \lambda_{\min}} = \frac{\lambda_{\max} - \alpha}{\lambda_{\max} + \alpha}$. It then follows (18) and (19).

Each step of the SAS iterative method is constituted of two-half steps which require finding solutions of linear equations with operators $(\alpha I + S)$ and $(\alpha I + A)$. Exact solutions of these equations are generally not available. These linear equations can be solved approximately using appropriate methods with respect to the properties of each operators. This results in the following inexact Self-adjoint/m-Accretive splitting (ISAS) iteration for solving the linear equation (1)-(2.

Given an initial guess $\bar{\psi}^{(0)} \in \mathcal{D}(T)$. For $k = 0, 1, 2, \ldots$ until $\{\bar{\psi}^{(k)}\}$ converges, solve $\bar{\psi}^{(k+\frac{1}{2})}$ approximately from

$$(\alpha I + S)\bar{\psi}^{(k+\frac{1}{2})} \approx (\alpha I - A)\bar{\psi}^{(k)} + q \tag{20}$$

by employing an inner iteration (e.g the conjugate gradient method) with $\bar{\psi}^{(k)}$ as the initial guess, then solve $\bar{\psi}^{(k+1)}$ approximately from

$$(\alpha I + A)\bar{\psi}^{(k+1)} \approx (\alpha I - S)\bar{\psi}^{(k+\frac{1}{2})} + q \tag{21}$$

by employing an inner iteration (e.g the MINRES method), where α is a given positive constant. The ISAS iteration method is proved to be convergent provide that the inner iterations converge [3].

Proposition 2.2 *In the case of isotropic scattering where $\kappa(r, \mu, \mu') = \sigma(r)c/2$ $(0 \leq c < 1)$,*

$$(\alpha I + S)^{-1} = \frac{1}{\sigma(r)(1 - c) + \alpha} P + \frac{1}{\sigma(r) + \alpha}(I - P); \quad P = \frac{1}{2}\int_{-1}^{1} \psi(r, \mu')d\mu' \tag{22}$$

Therefore, the two-step iteration method defined by (12) can be reduced to the following iteration: Given $\psi^{(0)} \in \mathcal{D}(T)$, for $k = 0, 1, ...$

$$(\alpha I + A)\psi^{(k+1)} = B\left[(\alpha I - A)\psi^{(k)} + q\right] + q, \tag{23}$$

where $B = \left(\frac{\alpha - \sigma(1-c)}{\alpha + \sigma(1-c)} - \frac{\alpha - \sigma}{\alpha + \sigma}\right) P + \frac{\alpha - \sigma}{\alpha + \sigma} I$.

proof. It follows from the fact that $P^2 = P$ and $(\alpha I + S) = \lambda(P + \epsilon(I - P))$ with $\lambda = \sigma(x)(1 - c) + \alpha$ and $\epsilon = \frac{\sigma(x) + \alpha}{\lambda}$.

3. An Implementation of the ISAS Method.

The ISAS iteration method can be implemented by employing a preconditioning technique. Let us consider the sequence $\{\psi^{(k)}\}_{k\in\mathbb{N}} \subset D(T)$, computed by the iteration (12). There exists a sequence $\{\varphi^{(k)}\}_{k\in\mathbb{N}} \subset L^2(Q)$, uniquely determined such that for $k = 0, 1, 2, \ldots$, $\psi^{(k)} = (\alpha I + A)^{-1}\varphi^{(k)}$. Since the operator $(\alpha I + A)^{-1}$ is bounded, if the sequence $\{\varphi^{(k)}\}_{k\in\mathbb{N}}$ converges to $\bar{\varphi}$, then $\{\psi^{(k)}\}_{k\in\mathbb{N}}$ will converge to $\bar{\psi} = (\alpha I + A)^{-1}\bar{\varphi}$. Substituting $\psi^{(k)}$ in (12), we obtain the following iteration: Given an initial guess $\varphi^{(0)} \in L^2(\Omega)$, for $k = 0, 1, \ldots$ until $\{\varphi^{(k)}\}$ converges, calculate

$$\left\{ \begin{array}{l} (\alpha I + S)\varphi^{(k+\frac{1}{2})} = (\alpha I - A)(\alpha I + A)^{-1}\varphi^{(k)} + q \\ \varphi^{(k+1)} = (\alpha I - S)\varphi^{(k+\frac{1}{2})} + q \end{array} \right. . \tag{24}$$

At each step k of the iteration method (24), we have to solve a linear system

$$\mathcal{A}(\alpha)F = q(\alpha), \tag{25}$$

where $\mathcal{A}(\alpha) = (\alpha I + S)$ and $q(\alpha) = (\alpha I - A)(\alpha I + A)^{-1}\varphi^{(k)} + q$. The solution of equation (25) is then used to compute $\varphi^{(k+1)}$. An infinite dimensional adaptation of the conjugate gradient method is employed to solve the equation (25). We have the following algorithm for the ISAS method:

Let $\psi^{(0)} \in \mathcal{D}(T)$, $\mathcal{R}^{(0)} = q - T\psi^{(0)}$, $q(\alpha) = (\alpha I - A)\psi^{(0)} + q$.

`While` $\|\mathcal{R}^{(k)}\| > \epsilon$ `do`

`begin`

`solve` $\mathcal{A}(\alpha)F = q(\alpha)$ `by CG method;`

`compute` $\varphi^{(k+1)} = (\alpha I - S)F + q$;

`compute` $q(\alpha) = (\alpha I - A)(\alpha I + A)^{-1}\varphi^{(k+1)}$;

`compute` $\mathcal{R}^{(k+1)} = q - (I - (\alpha I - S)(\alpha I + A)^{-1})\varphi^{(k+1)}$;

`end.`

In the previous algorithms, we have to make clear how the right hand side $q(\alpha)$ is computed, since it contains the inverse operator $(\alpha I + A)^{-1}$. Let $\varphi \in L^2(Q)$, we have $\phi = (\alpha I - A)(\alpha I + A)^{-1}\varphi = (\alpha I - A)f$. where $f \in \mathcal{D}(T)$ verifies the linear equation

$$(\alpha I + A)f = \varphi \tag{26}$$

Once f is calculated, ϕ can be easily computed. The differential equation (26) can be solved numerically.

4. Discretization and numerical results

Discretization. Let $\Omega = (0, R) \times (-1, 1)$. We consider the following triangulation of Ω:

$$\overline{\Omega} = \bigcup_{i,j}([r_i, r_{i+1}] \times [\mu_j, \mu_{j+1}]) = \bigcup_{i,j}\Omega_{i,j} \;\; 0 \le i \le N-1 \; and \; -J \le j \le J-1,$$

where $0 = r_0 < r_1 < \ldots < r_N = R$ and $-1 = \mu_{-J} < \mu_{-J+1} < \ldots < \mu_J = 1$ with $\mu_{-j} = -\mu_j$. We also consider the following nodes: $r_{i+\frac{1}{2}} = \theta_{i+\frac{1}{2}}r_{i+1} + (1 - \theta_{i+\frac{1}{2}})r_i$ and $\mu_{j+\frac{1}{2}} = \gamma_{j+\frac{1}{2}}\mu_{j+1} + (1-\gamma_{j+\frac{1}{2}})\mu_i$ where $\theta_{i+\frac{1}{2}} \in (0, 1)$ $(0 \le i \le N-1)$; $\gamma_{j+\frac{1}{2}} \in (0, 1)$ and $\gamma_{j+\frac{1}{2}} = 1 - \gamma_{-(j+\frac{1}{2})}$ $(-J \le j \le J-1)$. Let $U_{x,y}$ denotes the approximate value of the flux at (r_x, μ_y). We suppose that in Ω_{ij}, the approximate flux is a polynomial of degree ≤ 1 in r and μ. Therefore, we have the following relations:

$$\left\{ \begin{array}{l} \theta_{i+\frac{1}{2}}U_{i+1,j+\frac{1}{2}} + (1 - \theta_{i+\frac{1}{2}})U_{i,j+\frac{1}{2}} = U_{i+\frac{1}{2},j+\frac{1}{2}} \\ \gamma_{j+\frac{1}{2}}U_{i+\frac{1}{2},j} + (1 - \gamma_{j+\frac{1}{2}})U_{i+\frac{1}{2},j} = U_{i+\frac{1}{2},j+\frac{1}{2}} \end{array} \right. . \tag{27}$$

Using the DSN difference scheme [6], the discrete form of the equation (26) reads:

$$\left\{ \begin{array}{l} \eta_{j+\frac{1}{2}}(r^2_{i+1}U_{i+1,j+\frac{1}{2}} - r^2_i U_{i,j+\frac{1}{2}}) + \lambda_{i+\frac{1}{2}}(\beta_{j+1}U_{i+\frac{1}{2},j+1} - \beta_j U_{i+\frac{1}{2},j})+ \\ \alpha\nu_{i+\frac{1}{2},j+\frac{1}{2}}U_{i+\frac{1}{2},j+\frac{1}{2}} = \nu_{i+\frac{1}{2},j+\frac{1}{2}}f_{i+\frac{1}{2},j+\frac{1}{2}} \\ 0 \le i \le N-1; -J \le j \le J-1 \end{array} \right. , \tag{28}$$

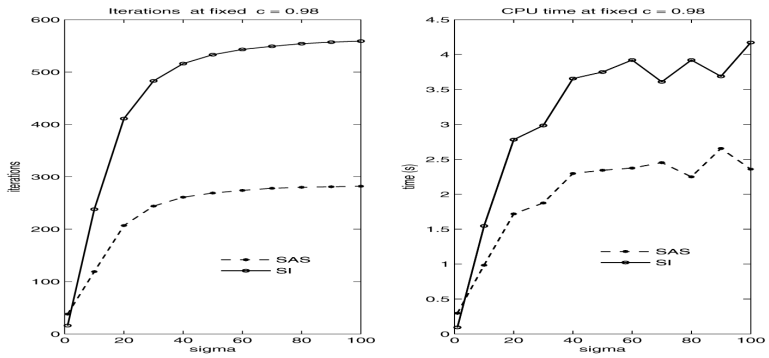

Figure 1: Comparison of the SI and SAS methods at fixed $c = 0.98$ ($\epsilon = 10^{-5}$): (left) number of iterations; (right) CPU time.
where $\eta_{j+\frac{1}{2}} = \omega_{j+\frac{1}{2}}\mu_{j+\frac{1}{2}}$, $\lambda_{i+\frac{1}{2}} = w_{i+\frac{1}{2}}r_{i+\frac{1}{2}}$, $\nu_{i+\frac{1}{2},j+\frac{1}{2}} = \omega_{j+\frac{1}{2}}w_{i+\frac{1}{2}}r^2_{i+\frac{1}{2}}$ and

$$\left\{\begin{array}{l} r^2_{i+1} - r^2_i = 2w_{i+\frac{1}{2}}r_{i+\frac{1}{2}}, \quad r^3_{i+1} - r^3_i = 3w_{i+\frac{1}{2}}r^2_{i+\frac{1}{2}} \\ \beta_{j+1}\mu_{j+1} - \beta_j\mu_j = \omega_{j+\frac{1}{2}}(1 - 3\mu^2_{j+\frac{1}{2}}), \quad \beta_{j+1} - \beta_j = -2\mu_{j+\frac{1}{2}}\omega_{j+\frac{1}{2}} \\ \omega_{j+\frac{1}{2}} = \mu_{j+1} - \mu_j, \quad \beta_J = 0 \end{array}\right. , \qquad (29)$$

The system (28) is completed with the linear system obtained from discretization of the equation (26) at $\mu = -1$ and the symmetry condition at $r = 0$. The resulting system is solved explicitly to obtain $U_{i,j+\frac{1}{2}}$, $U_{i+\frac{1}{2},j}$ and $U_{i+\frac{1}{2},j+\frac{1}{2}}$.

Numerical results. We present numerical results from the application of the ISAS method on an example problem. We took particular data for which an exact solution u is known: $\sigma(r) = \sigma$, $\kappa(r, \mu, \mu') = \frac{\sigma c}{2}$,$(0 < c < 1)$, $R = 1$

$$q(r, \mu) = \sigma(1 - c)(1 - r) - \mu; \quad u(r, \mu) = 1 - r$$

and we compared the speed (number of iteration and CPU time) of ISAS and standard SI algorithms at fixed c and at fixed σ. For iterative methods tested here, the iterations are stopped when the relative error $\frac{\|U-U^{(k)}\|_2}{\|U\|_2}$ is less than a prescribed ϵ. For this problem, the theoretical value of the SAS iteration optimal parameter is $\alpha_t = \sigma(1 - c)$. For the ISAS iteration, we set $\alpha = \sigma$. The spatial mesh size is $h = 1/500$ and the angular mesh size is $\tau = 1/10$.

The Figure 1 represents as function of σ $(1 \leq \sigma \leq 100)$ the number of iteration and the CPU time at $c = 0.98$ corresponding the SI and ISAS algorithm for $\epsilon = 1E - 05$. The Figure 2 represents as function of c $(0 < c < 1)$ the number of iteration and the CPU time at $\sigma = 100$ corresponding the SI and ISAS algorithm for $\epsilon = 1E - 05$. The Figure 3 represents as function of σ (for large σ) the number of iteration and the CPU time at $c = 0.98$ corresponding the SI and ISAS algorithm for $\epsilon = 1E - 05$. The Figure 4 represents as function of c ($c \approx 1$) the number of iteration and the CPU time at $\sigma = 50$ corresponding the SI and ISAS algorithm for $\epsilon = 5E - 04$. The Figure 5 represents the convergence rate as function of the number of iteration at $c = 0.5$ $(\sigma \in \{10, 100, 500\})$ and at $c = 1$ $(\sigma \in \{10, 50, 100\})$. It appears from these tests that the ISAS algorithm is efficient compare to SI algorithm.

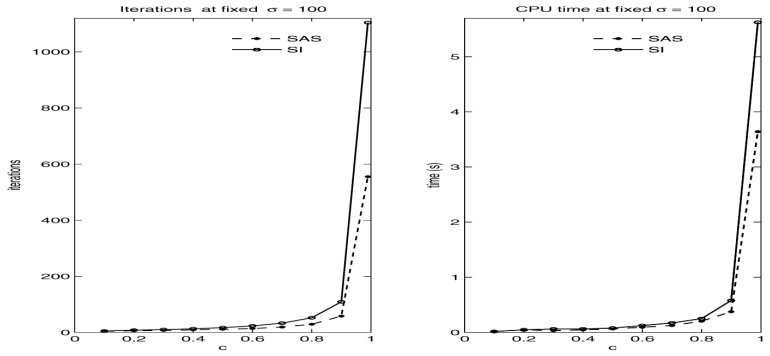

Figure 2: Comparison of the SI and SAS methods at fixed $\sigma = 100$ ($\epsilon = 10^{-5}$): (left) number of iterations; (right) CPU time.

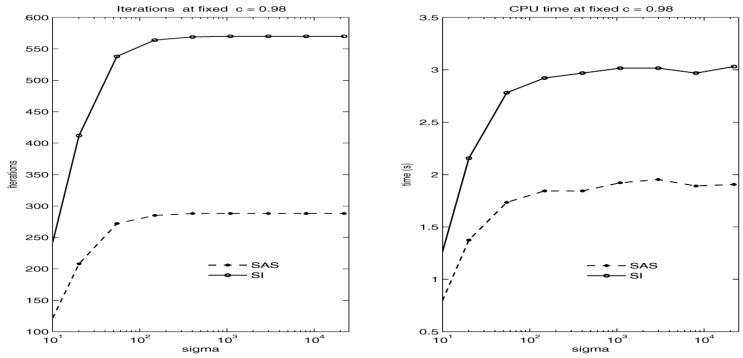

Figure 3: Comparison of the SI and SAS methods at fixed $c = 0.98$, for large σ ($\epsilon = 10^{-5}$): (left) number of iterations; (right) CPU time.

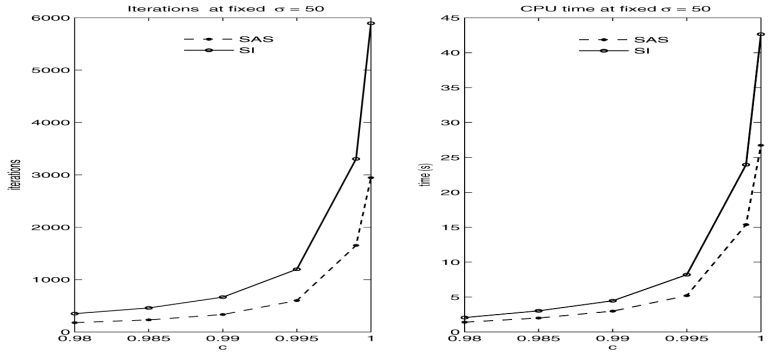

Figure 4: Comparison of the SI and SAS methods at fixed $\sigma = 50$, for $c \approx 1$ ($\epsilon = 5E-04$): (left) number of iterations; (right) CPU time.

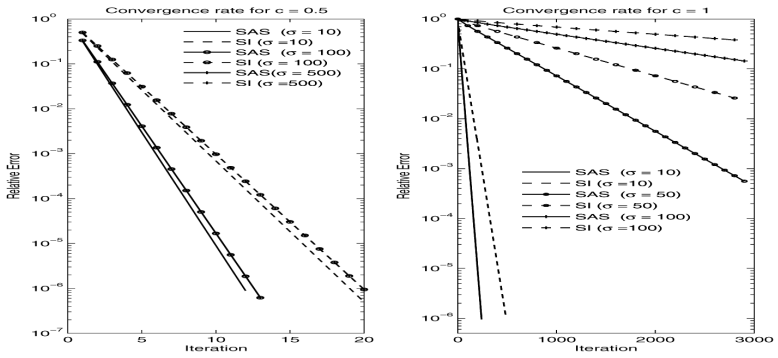

Figure 5: Comparison of the convergence rate at fixed c of the SI and SAS for several values of σ : (left) $c = 0.5$; (right) $c = 1$.

5. Conclusion

Throughout this work, it comes that the iterative methods based on a Self-adjoint and m-Accretive splitting presented for solving the transport equation in 1D-spherical geometry, converge unconditionally. The theoretical proof of the convergence of the method is independent of the discretization. The previous numerical results show that the SAS iteration is efficient compare to the standard Source Iteration. The method is easy to implemented as SI method.

6. References

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