Markov analysis of land use dynamics
A Case Study in Madagascar

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RÉSUMÉ. Nous présentons une modélisation markovienne d'une dynamique d'usage des sols le long d'un corridor forestier de Madagascar. Ces zones géographiques concentrent des enjeux écologiques et économiques de première importance. Nous disposons d'un jeu de données sur 22 ans et 42 parcelles, chaque parcelle pouvant prendre 4 états. L'état initial est écarté et nous proposons un modèle de Markov à 3 états. Un premier traitement des données par maximum de vraisemblance conduit à un modèle admettant un état absorbant. Nous étudions alors la loi quasi-stationnaire du modèle et la loi du temps d'atteinte de l'état absorbant. Selon les experts, une transition non présente dans les données doit être rajoutée au modèle nous amenant à utiliser une approche bayésienne afin identifier le modèle. Nous obtenons alors un modèle régulier admettant une loi stationnaire. Nous étudions la vitesse de convergence vers cet équilibre. Enfin nous analysons la dynamique ainsi identifiée. Les deux approches, sur une échelle de temps réaliste, conduisent à des résultats cohérents.

ABSTRACT. We present a Markov model of a land-use dynamic along a forest corridor of Madagascar. These geographic spots are of primary ecological and economic importance. We have a data set of 22 years on 42 plots, each plot can take four states. The initial state is removed and we use a Markov model with three states. A initial processing of data by the maximum likelihood approach leads to a model admitting an absorbing state. We study the quasi-stationary distribution law of the model and the law of the hitting time of the absorbing state. According to experts, a transition not present in the data, must be added in the model leading to a Bayesian approach to identify the model. We obtain a regular model admitting an invariant distribution law. We study the speed of convergence to equilibrium. Finally we analyze the two dynamics that have been identified, on a realistic time scale, leading to consistent results.

MOTS-CLÉS : Modèle de Markov; dynamique d'usage des sols; méthode de Monte Carlo par chaîne de Markov; lois stationnaire et quasistationnaire.

KEYWORDS : Markov model; land use dynamics ; Markov chain Monte Carlo; stationary and quasi stationary distribution laws

Population pressure is one of the major causes of deforestation in tropical countries. In the region of Fianarantsoa (Madagascar), two national parks Ranomafana and Andringitra are connected by a forest corridor, which is of critical importance to maintain the regional biodiversity, see Figure 1. The need for cultivated land pushes people to encroach on the corridor. To reconciling forest conservation with agricultural production, it is important to understand and model the dynamic of post-forest land use of these parcels.

We will use a data set developed by IRD, see Figure 1, corresponding to 42 parcels and 22 years, from 1985 to 2006. The states are : annual crop $(\mathtt{A})$, fallow $(\mathtt{F})$, perennial crop $(\mathtt{P})$ and natural forest $(\mathtt{f})$. The $\mathtt{f}$ state will be omitted in the final observation series as it is transient and it provides no information. The $\mathtt{f}$ state is systemically followed by the $\mathtt{A}$ state that will be considered as the first observation. The observation series will be denoted $(e^{(p)}_{1:N_p})_{p=1:42}$ where $e^{(p)}_n$ belongs to the state space $E \stackrel{\text{def}}{=} \{\mathtt{A},\mathtt{F},\mathtt{P}\}$ and N_p is the length of the observation series of the parcel p. Here the notation $n = n_1 : n_2$ stands for $n = n_1, \dots, n_2$ for $n_1 \leq n_2$.

1. A first model derived from the maximum likelihood estimate

We make the following hypothesis :

(H_1) *The dynamics of the parcels are independent and identical, they are Markovian and time-homogeneous.*

This means that $(e^{(p)}_{1:N_p})_{p=1:42}$, are 42 independent realizations of a same process $(X_n)_{n\geq 0}$ and that this process is Markov and time-homogeneous. This assumption is of course simplistic as the dynamics of a given parcel may depends on : farmer decisions ; exposition, slope and distance from the forest, that means properties of the same plot ; neighboring parcels. Assumption (H_1) however will prove interesting in this first study, it will allow us to build a simple model that nevertheless lead us to interesting results.

To identify the 3×3 transition matrix Q (6 parameters) we first use the maximum likelihood approach, this consists simply in calculating the empirical transition matrix, i.e. the the relative occurrences of all the transitions (see [3] for details) :

$$\hat{Q}_{ij} = \#\{i \to j\} / \textstyle\sum_{j' \in E} \#\{i \to j'\}$$

where $\#\{i \to j\}$ is the number of transitions from i to j in the dataset. The resulting matrix is depicted as graph in Figure 2 (left : model 1).

Naturally as the transitions $\mathtt{PA}$, $\mathtt{FP}$ and $\mathtt{PF}$ are not in the initial data set, they do not appear in the model. The main implication is that the state $\mathtt{P}$ is absorbing and the other states are transient so that the limit distribution is $\delta_{\mathtt{P}}$. So a first question is to describe the

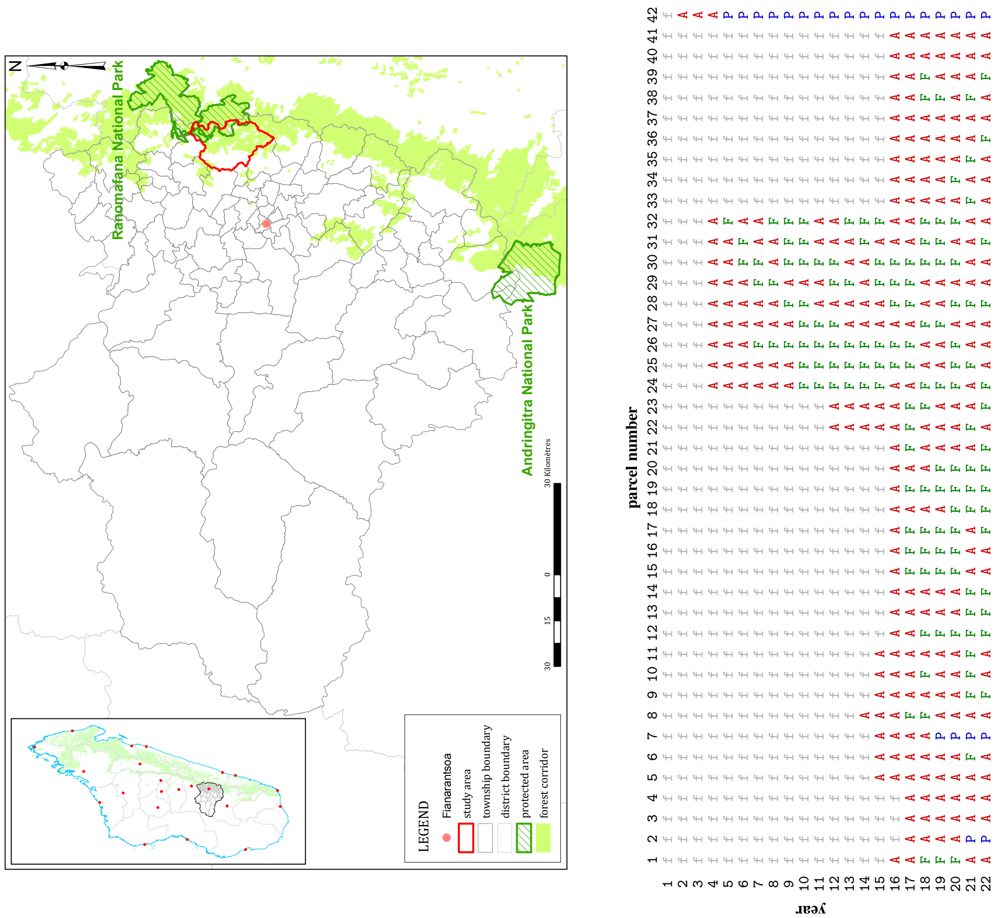


Figure 1 – *Left : The study area bordering the forest corridor between Ranomafana and Andringitra. Right : Annual states corresponding to 42 parcels and 22 years. These data were collected between the years 1985 and 2006. The parcels are located on the slopes and lowlands on the edge of the forest corridor of Ranomafana-Andringitra, Madagascar, see Figure 1. The states are : annual crop* (`A`)*, fallow* (`F`)*, perennial crop* (`P`) *and natural forest* (`f`)*. The* `f` *state, that will be omitted, is systemically followed by the* `A` *state that will be considered as the first observation. Hence the observation series* $(e^{(p)}_{n=1:N_p})_{p=1:42}$ *will be the sequence of states* $\{\mathtt{A}, \mathtt{F}, \mathtt{P}\}$ *and* N_p *will be the length of the observation series of the parcel* p*, e.g.* $N_1 = 7$*,* $N_2 = 6$ *etc.*

behavior of the process before it reaches P and a second question is to analyze the time length to reach the state P, that is the distribution law of $\tau_{\mathtt{P}}$ where :

$$\tau_e \stackrel{\text{def}}{=} \inf\{n \geq 1 \,:\, X_n = e\} \text{ if } \exists n \geq 1 \text{ such that } X_n = e,\ +\infty \text{ otherwise}. \qquad (1)$$

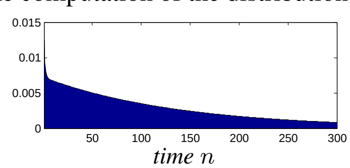
is the first time to reach a given state e.

The answer to the first question is given by the so-called quasi-stationary distribution, it is roughly the "limit" distribution that the system reach before being absorbed by P. We consider the probability to be in $e \in \{\mathtt{A},\mathtt{F}\}$ before reaching P and starting from A, i.e. $\mu_n(e) = \mathbb{P}(X_n = e | \tau_{\mathtt{P}} > n,\ X_0 = \mathtt{A})$ where $\tau_{\mathtt{P}}$ is the first time to reach P. Suppose that $\mu_n(e) \to \sigma_{\text{qs}}(e)$ as $n \to \infty$ for $e \in \{\mathtt{A},\mathtt{F}\}$ then the probability distribution $(\sigma_{\text{qs}}(e))_{e\in\{\mathtt{A},\mathtt{F}\}}$ is called quasi-stationary probability distribution. This problem was originally solved in [6] : $\sigma_{\text{qs}} = [\sigma_{\text{qs}}(\mathtt{A}), \sigma_{\text{qs}}(\mathtt{F})]$ exists and is solution of

$$\sigma_{\text{qs}}\, Q_{\text{qs}} = \lambda\, \sigma_{\text{qs}}\,, \qquad\qquad \text{with } Q = \left(\begin{array}{c|c} Q_{\text{qs}} & q \\ \hline 0 & 1 \end{array}\right) \qquad (2)$$

with $\sigma_{\text{qs}}(e) \geq 0$, $\sigma_{\text{qs}}(\mathtt{A}) + \sigma_{\text{qs}}(\mathtt{F}) = 1$, and λ is the spectral radius of Q_{qs}. In (2), Q_{qs} is the submatrix of Q corresponding to the 2 first states.

From (2) we can compute the quasi-stationary distribution σ_{qs} associated with the MLE $\hat{Q}$ of Q. We get : $\sigma_{\text{qs}} = (\sigma_{\text{qs}}(\mathtt{A}), \sigma_{\text{qs}}(\mathtt{F})) = (0.5794, 0.4206)$. Hence, conditionally to the fact that the process does not reach P, it will spend 58% of its time in the A state and 42% of its time in the F state. The computation of the distribution law of $\tau_{\mathtt{P}}$ is explicit, we get :



The mean time to reach P is 136 years with a standard deviation of 135 years.

2. A second model derived from a Bayesian approach

The model derived from the maximum likelihood estimate does not allow the transitions PA, FP and PF. There is no logic for the transitions FP and PF, but the transition PA can be observed on a time scale of several decades. So we suppose that :

(H_2) *The transitions* FP *and* PF *are not possible, all other transitions are possible.*

This hypothesis leads to a transition matrix Q with four parameters $\theta = (\theta_i)_{i=1,2,3,4}$.

The maximum likelihood approach will lead to the model of Figure 2 (left) where $Q(\mathtt{P},\mathtt{A}) = 0$. We therefore have to make use of Bayesian methods. Given a prior distribution law $\pi(\theta)$ on the parameter θ, According to the Bayes rule, the posterior distribution law $\pi(\theta)$ on θ given the observations X is :

$$\pi_{\text{post}}(\theta) \propto L_2(\theta)\ \pi_{\text{prior}}(\theta) \tag{3}$$

("$\propto$" means that $\pi_{\text{post}}(\theta) = L_2(\theta)\ \pi_{\text{prior}}(\theta)/\int L_2(\theta')\ \pi_{\text{prior}}(\theta')\,\mathrm{d}\theta'$) where $L_2(\theta)$ is the likelihood function, let $l_2(\theta)$ be the corresponding log-likelihood function. The derivation of these expressions are straightforward, details can be found in [3]. The Bayes estimator $\tilde{\theta}$ of the parameter θ is the mean of the a posteriori distribution :

$$\tilde{\theta} \stackrel{\text{def}}{=} \int_\Theta \theta\, \pi_{\text{post}}(\theta)\,\mathrm{d}\theta = \frac{\int_\Theta \theta\, L_2(\theta)\, \pi_{\text{prior}}(\theta)\,\mathrm{d}\theta}{\int_\Theta L_2(\theta)\, \pi_{\text{prior}}(\theta)\,\mathrm{d}\theta}\,. \tag{4}$$

Numerical tests have been performed and suggest that the Jeffreys prior is well adapted to the present situation. The Jeffreys prior distribution (non-informative) is defined by :

$$\pi_{\text{prior}}(\theta) \propto \sqrt{\det[\mathcal{I}(\theta)]} \qquad \text{where } \mathcal{I}(\theta) \stackrel{\text{def}}{=} \Big[\mathbb{E}_\theta\Big(-\frac{\partial^2 l_2(\theta)}{\partial\theta_k\,\partial\theta_l}\Big)\Big]_{1\le k,l\le 4}\,, \tag{5}$$

$\mathcal{I}(\theta)$ is the Fisher information matrix. Again, see [3] for a precise expression of $\pi_{\text{prior}}(\theta)$.

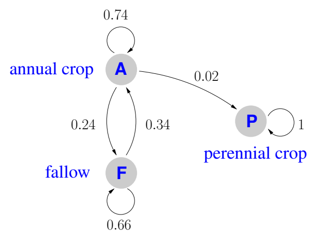
Although the Jeffrey prior distribution is explicit, we cannot compute analytically the corresponding Bayes estimator (4). We use a Monte Carlo Markov chain (MCMC) method, namely a Metropolis-Hastings algorithm with a Gaussian proposal distribution.

For the real data, the Bayes estimation of the transition matrix with the Jeffreys prior and the MCMC method leads to the transition matrix $\tilde{Q}$ depicted as a graph in Figure 2 (right : model 2). Note that the probability of the transition `PA` is now strictly positive. Moreover, as $[\tilde{Q}^2]_{ij} > 0$ for all i, j, $\tilde{Q}$ is a regular transition matrix (i.e. irreducible and aperiodic) and so there exist a unique invariant measure σ solution of $\sigma = \sigma\,\tilde{Q}$. After computation : $\sigma = (0.4127\,,\ 0.3065\,,\ 0.2808)$.

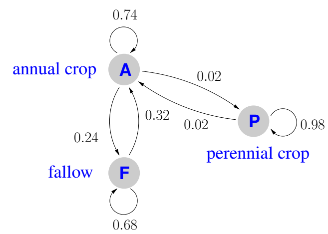
3. Discussion

We derived two Markovian models, see Figure 2. In the first model, `P` is an absorbing taste and the limit distribution will charge this state only. Model 2 is regular and admits a limit invariant measure which is strictly positive for the 3 states. The difference is that for the first model the probability of the transition `PA` is null and it is 0.02 for the second model. Starting from the state `A`, the evolution of the proportions of parcels in state `A`, `F` and `P` for the two models is given in Figure 3 (left).

These evolutions are almost identical in an horizon of 20 years, they appear to be different only after few decades. They are radically different on the long scale of centuries : for model 1 almost all parcels are in the state `P`, model 2 converges to an equilibrium after 100 years. In the first model we saw that the mean time to reach the state `P` is 136 years.



Model 1 (Maximum likelihood estimation)



Model 2 (Bayesian estimation)

Figure 2 – *Markov models derived by the direct maximum likelihood approach (left) and the Bayesian approach with a Jeffreys prior.*

In the second model, the equilibrium is reached after one century and it is 41% of parcels in state `A`, 31% in state `F`, 28% in state `P`.

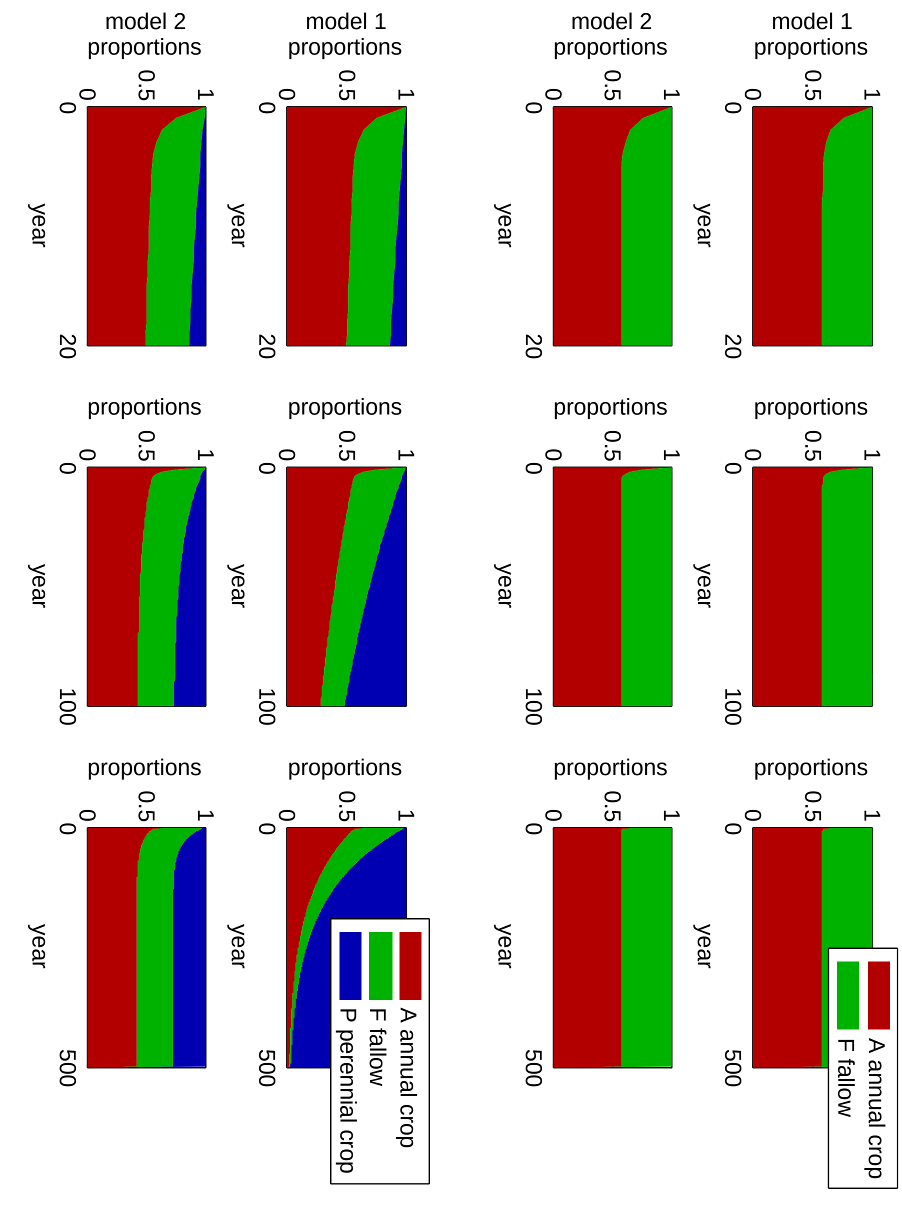
Model 1 presents a quasistationary behavior. Indeed if we consider the evolution of the relative proportions of parcels in state `A` and in state `F`, model 1 and model 2 present quasi identical profiles, see Figure 3 (right). Hence after less than 5 years, the relative proportions of parcels in state `A` and in state `F` is close to 58% and 42%.

In [3] we assessed the adequacy of the model to real data. We tested if the empirical sojourn times correspond to a geometric distribution. We used a parametric bootstrap goodness-of-fit on empirical distribution.

Not that in the Model 2, the probabilities of transition `AP` and of transition `PA` are both equal to 0.02, still the variance on the estimation of the transition probability `AP`, which appears in the data set, is smaller than the variance on the estimation of the transition probability `PA` see details in [4].

4. Perspectives

A new database is currently being developed by the IRD. It will be for a longer period of time and a greater number of parcels, it will also allow to consider a more detailed state space comprising more than three states. Part of the complexity of these agro-ecological temporal data comes from the fact that some transitions are "natural", due to ecological dynamics, while others come from human decisions (annual cropping, crop abandonment, planting perennial crops, etc.). It should also be interesting to study the dynamics of parcels conditionally on the dynamics of the neighbor parcels. This model could be more realistic but requires first studying the farmers' practices in order to limit the number of unknown parameters in the model. Such spatio-temporal models taking into account the neighborhood dynamics have been already proposed, notably in the context of deforestation in Madagascar [1].



3 states

restriction to the states `A` and `F`

Figure 3 – *Evolution of the proportions of parcels in state* `A`, `F` *and* `P` *for the two models. Left : with the three states ; right : restricted to the two first states.*

The present results are preliminary, still they allow to prove that the data set time scale permits to infer the addressed questions on the corridor dynamics. In the near future we will relax the Markov hypothesis and consider semi-Markov models.

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